Localization and the interface between quantum mechanics, quantum field theory and quantum gravity

dedicated to the memory of Rob Clifton

submitted to "Studies in History and Philosophy of Physics"

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Abstract

We show that there are significant conceptual differences between QM and QFT which make it difficult to view the latter as just a relativistic extension of the principles of QM. At the root of this is a fundamental distiction between Born-localization in QM (which in the relativistic context changes its name to Newton-Wigner localization) and *modular localization* which is the localization underlying QFT, after one liberates it from its standard presentation in terms of field coordinates. The first comes with a probability notion and projection operators, whereas the latter describes causal propagation in QFT and leads to thermal aspects of locally reduced finite energy states. The Born-Newton-Wigner localization in QFT is only applicable asymptotically and the covariant correlation between asymptotic in and out localization projectors is the basis of the existence of an invariant scattering matrix.

Taking these significant differences serious has not only repercussions for the philosophy of science, but also leads to a new structural properties as a consequence of vacuum polarization: the area law for *localization entropy* near the the causal localization horizon and a more realistic cutoff independent setting for the cosmological vacuum energy density which is compatible with local covariance. The article presents some observations about the interface between QFT in CST and QG.

1 Introductory remarks

Ever since QM was discovered, the conceptual differences between classical theory and quantum mechanics (QM) have been the subject of fundamental investigations with profound physical and philosophical consequences. But the conceptual relation between quantum field theory (QFT) and QM, which is at least as challenging and rich of surprises, has not received the same amount of attention and scrutiny, and often the subsuming of QFT under "relativistic QM" nourished prejudices and prevented a critical foundational debate. Apart from some admirable work on the significant changes which the theory of measurements must undergo in order to be consistent with the structure of QFT [1] and some deep mathematical related work related to it [2], the knowledge on this subject has remained in the mind of a few individuals working on the foundations of QFT.

Often results of this kind which involve advanced knowledge of QFT do not attract much attention even when they have bearings on the foundations of QT as e.g. the issue of *Bell states* in *local quantum physics* (LQP[1]) [4]

[1]We use this terminology instead of QFT if we want to direct the reader's attention away from the textbook Lagrangian quantization towards the underlying principles [3]. QFT (the content of QFT textbooks) and LQP deal with the same physical principles but LQP is less comitted to a particular formalism (Lagrangian quantization, functional integrals) and rather procures always the most adaequate mathematical concepts for their implementation. It includes of course all the results of the standard perturbative Lagrangian quantization

or the important relations between causal disjointness with the existence of uncorrelated states as well as the issue to what extend causal independence is a consequence of statistical independence [5]. The reason is not so much a lack of interest but rather that QFT is often thought to be just a kind of relativistic quantum mechanics. This may explain why there has been a tremendous effort on the side of quantum mechanical foundations and very little investment on the side of QFT. Indeed there is an amazing lack of balance between the very detailed and sophisticated literature about interpretational aspects of QM and its relation with information theory, aiming sometimes at some very fine, if not to say academic/metaphoric points (e.g. the multiworld interpretation), and the almost complete lack of profound interpretive activities about our most fundamental theory of matter. Although the name QT usually appears in the title of foundational papers, this mostly hides the fact that they deal exclusively with concepts from QM leaving out QFT.

If on the other hand some foundational motivated quantum theorist become aware of the deep conceptual differences between particles and fields they tend to look at them as antagonistic and create a battleground; the fact that they are fully compatible where for physical reasons they must agree, namely in the asymptotic region of scattering theory, is usually overlooked.

The aim of this essay to show that at the root of these differences there are two localization concepts: the quantum mechanical Born-Newton-Wigner localization and the modular localization of LQP. The B-N-W localization is not Poincaré covariant but attains this property in a certain asymptotic limit namely the one which is needed in scattering theory. Modular localization on the other hand is causal at all distances but lacks projectors on subspaces, the linear spaces of localized states are usually dense in the Hilbert space of all states. One of the aims of this article is to collect some facts which show that besides sharing the notion of Hilbert space, operators and states as well as $\hbar$, QM and QFT are conceptually worlds apart and yet they harmonize perfectly in the asymptotic region of scattering theory.

In this connection one is reminded that some spectacular misunderstanding of conceptual properties in passing from QM to LQP led to incorrect results about alleged violations of the velocity of light remaining a limiting velocity in the quantum setting (the famous Fermi Gedankenexperiment). As a result of a publication in Phys. Lett. [6] and a simultaneous article in *Nature* on the prospects of time machines, this created quite a stir at the time and led to a counter article [7]. Since the LQP presentation of the Fermi Gedankenexperiment has been a strong motivations for non-experts to engage with its conceptual setting, and hence has some pedagogical merits in the present context, it is natural that it will also obtain some space in this article. Although these kinds of sophisticated misunderstanding continue to appear occasionally in papers, only the mentioned episode made it into the world press (as a result of the impact of PLR articles on popular scientific journals as Nature and on

but presents them in a conceptually and mathematically more satisfactory way. Most of the subjects in this article are outside of textbook QFT.

the international press.)

It is not our intention to present a new axiomatic setting (for an older presentation see [3]). Such a goal would be too ambitious in view of the fact that we are confronting a theory where, in contradistinction to QM, no conceptual closure is yet in sight. Although there has been some remarkable nonperturbative progress concerning constructive control (i.e. solving the existence problem) of models, the main knowledge about models of QFT is still limited to numerically successful but nevertheless diverging perturbative series.

Here the more modest aim is to collect some either unknown or little known facts which could present some food for thoughts about a more inclusive measurement theory, including all of quantum theory (QT) end not just QM. On the other hand one would like to improve the understanding about the interface between QFT in CST (curved spacetime) and the still elusive QG. This can only be achieved by going somewhat beyond the presently fashionable "shut up and calculate" attitude. But if one has to enter speculative excursions one would like to do this from a solid conceptual-mathematical platform, so in case the trip into the unknown ends in nowhere, there is a return and/or a chance to modify the direction[2].

Since both expressions QFT and LQP are used do denote the same theory, let me emphasize again that there is no difference in content between; LQP is used instead of QFT whenever the conceptual level of the presentations gets beyond that which the reader is able to find in standard textbooks of QFT. There is of course one recommendable exception, namely Rudolf Haag's book "Local Quantum physics" [3]; but in a fast developing area of particle physics two decades (referring to the time it was written) are a long time.

The paper consists of two main parts, the first is entirely dedicated to the exposition of the differences between (relativistic[3]) QM and LQP, whereas the second deals with thermal consequences of vacuum polarization caused by causal localization and some consequences for QFT in curved spacetime (CST). A QG theory does not yet exist, but a profound understanding of those foundational aspects are expected to be important to get there.

The first part starts with a subsection on *direct particle interactions* (DPI), a framework which incorporates all those properties of a relativistic theory which one is able to formulate solely in terms of relativistic particles; some of them already appearing in the S-matrix work of E.C.G. Stückelberg. However the enforcement of the cluster factorization property (the spatial aspect of macrocausality) in DPI requires more involved arguments. It is not automatic as in nonrelativistic QM where it follows from the additivity of interaction term. As a result DPI does not allow a second quantization presentation, even though it is a perfect legitimate multiparticle theory in which n-particles are linked to n+1

[2]In my lifetime I have seen 3 TOEs (theories of everything) fail and a fourth is already in an intensive care unit. It is common to all these attempts at a TOE that none of them were started from a solid conceptual platform and only a few of their fans succeeded to return to secure areas of particle physics.

[3]In order to show that making QM relativistic does not remove the fundamental differences with QFT the next section will be on the relativistic setting of "direct particle interactions".

particles by cluster factorization. Within the particle physics community there seems to be a lack of awareness about its existence which may be due to the fact that its protagonists are theoretical nuclear physicists who wanted to construct a relativistic particle theory for an intermediate energy range for which relativistic invariance is already important but only a few particles can be created. Most particle physicists tend to believe that a relativistic particle theory, consistent with macro-causality and a Poincaré-invariant S-matrix, must be equivalent to QFT[4], therefore it may be helpful to show that this is not correct.

Since the ideas which go into its construction are important for appreciating the conceptual differences of relativistic QM to QFT, we will at least sketch some of the arguments showing that DPI theories fulfill all the physical requirements which one is able to formulate solely in terms relativistic particles without recourse to fields, as Poincaré covariance, unitary and macro-causality of the resulting S-matrix (which includes cluster factorization). In contradistinction to nonrelativistic mechanics for which clustering follows trivially from the additivity of pair-(or higher-) particle potentials, and also in contradistinction to QFT where the clustering is a rather straightforward consequence of locality and the energy positivity, the implementation in the relativistic DPI setting is much more subtle on this point (and this is related to the lack of a second quantization reformulation of multi-particle interactions in such theories). The important point in the present context is that there exists a quantum mechanical relativistic setting in which the S-matrix is Poincaré invariant, fulfills macro-causality and implements interaction without using fields.

In this way one learns to appreciate the fundamental difference between quantum theories which have no maximal velocity and those which have. As a quantum mechanical theory DPI only leads to statistical "effective" finite velocity propagation for asymptotically large time-like separations between localized events as they occur in scattering theory. With other words the causal propagation between Born-localized events is recovered only in the sense of asymptotically large timelike distances. This explains in particular why in such theories the S-matrix is Poincaré invariant. Saying that DPI is macro- but not micro-causal implies that it cannot be used to study properties of local propagation over finite distances. Asymptotically Fermi's Gedankenexperiment leads to the desired result in QM (DPI) and QFT. But only in the different notion of causal propagation which is totally characteristic for QFT and does not exist in DPI. For relativistic scattering theory be it DPI or QFT, the projectors and the related probablities which come with B-N-W localization are indispensible.

So at the root of the alleged QM-QFT (particle-field) antagonism is the existence of two very different concepts of localization namely the *Born localization* which is the only localization for QM, and the *modular localization* which is the one underlying the causal locality notion in QFT together with Born localization. the two only coalesce for infinite timelike separations of events. For scattering theory in any relativistic theory be it DPI or QFT one needs the

[4] The related folklore one finds in the literature amounts to the dictum: relativistic quantum theory of particles + cluster factorization property = QFT. Apparently this conjecture goes back to S. Weinberg.

Born-localization. In fact one glance into the original paper by Born reveals that the probability interpretation was made on scattering amplitudes leading to what is nowadays called cross section in the Born approximation, the x-space wave functions; on the other hand without modular localization there would be no interaction-induced vacuum polarization and instead the world at finite distances would be filled with little acausal poltergeist-daemons.

Whereas QM only knows the Born localization, QFT requires both, Born-localization for (the wave functions of) particles before and after a scattering event, and modular localization[5] in connection with fields and local observables[6]. Without Born localization and the associated projectors, there would be no scattering theory leading to cross sections and QFT would become just a mathematical playground.

In contradistinction to DPI, in interacting QFT there is no way in which in the presence of interactions the notion of *particles at finite times* can be saved. The statement that an isolated relativistic particle cannot be localized below its Compton wave length refers to the (Newton-Wigner adaptation of the) Born localization and is meant, as all statements involving Born localization, in an *effective* probabilistic sense. Only in the timelike asymptotic limit between two Born localization events, sharp geometric relations with c being the maximal velocity emerge; this is precisely what one needs to obtain a Poincaré invariant macrocausal S-matrix. The maximal velocity in the sense of asymptotic expectations in suitable states is of course the same mechanism which in nonrelativistic QM leads to material-dependent acoustic velocities.

The first part focusses on the radical difference between the Newton-Wigner (NW) localization (the name for the Born localization after the adaptation to the relativistic particle setting) and the localization which is inherent in QFT, which in its intrinsic form, i.e. liberated from singular pointlike "field coordinatizations", is referred to as *modular localization* [9][11][10]. The terminology has its origin in the fact that it is backed up by a mathematical theory within the setting of operator algebras which bears the name Tomita-Takesaki[7] *modular theory*. Within the setting of thermal QFT, physicists independently discovered various aspects of this theory [3]. Its relevance for causal localization was only spotted a decade later [14] and the appreciation of its use in problems of thermal behavior at causal- and event- horizons and black hole physics had to wait another decade [15].

The last subsection of the first part presents LQP as the result of *relative positioning* of a finite (and rather small) number of *monads* within a Hilbert space. This shows the enormous conceptual distance between QM and LQP. Here we are using a terminology which Leibniz introduced in a philosophical-

[5] Modular localization is the same as the causal localization inherent in QFT after one liberates the letter from the contingencies of particular selected fields.

[6] Particles are objects with a well-defined ontological status, whereas (composite) fields form an infinite set of coordinatizations which generate the local algebras. Modular localization is the localization property which is independent of what field coordinatization has been used.

[7] Tomita was a Japanese mathematician who discovered the main properties of the theory in the first half of the 60s, but it needed a lot of polishing in order to be accepted by the mathematical community, and this is where the name Takesaki entered.

ontological context. Whereas a single monad also appears in different contexts e.g. the information theoretical interpretation of bipartite spin algebras in suitable singular states [2][16], the modular positioning of several copies is totally characteristic for LQP. Although its physical and mathematical content is quite different from Mermin's [17] new look (the "Ithaca-interpretation" of QM) at quantum mechanical reality exclusively in terms of correlations between subsystems, they share the aspect of understanding reality in relational terms. Mathematically a monad in the sense of this article is the unique hyperfinite type III_1 factor algebra to which all local algebras in LQP are isomorphic, so all concrete monads are copies of the abstract monad. Naturally a monade has no structure per se, the reality emerges from their relation to each other.

Whereas for Newton physical reality consisted of matter moving in a fixed space according to a universal time, reality for Leibniz emerges from interrelations between monads with spacetime serving as ordering device. The modular positioning of monads goes one step further in that even the Minkowski spacetime together with its invariance group the Poincaré group appears as a consequence of positioning in a more abstract sense namely of a finite number of monads in a joint Hilbert space (subsection 7). For actual constructions of interacting LQP models it is however advantageous to start with one monad and the action of the Poincaré group on it.

The algebraic structure of QM on the other hand, relativistic or not, has no such monad structure; the global algebra as well as all Born-localized subalgebras in ground states are always of type I i.e. either the algebra of all bounded operators $B(H)$ in an appropriate Hilbert space or multiples thereof. Correlations are characteristic features of quantum mechanical states, whereas for the characterization of a QM system global operators as the Hamiltonian are indispensable.

The second part addresses two important consequences of vacuum polarization, the first subsection deals with *localization entropy* and recalls its area proportionality which is a more recent result [65][63]. The thermal aspects of localization have astrophysical and cosmological consequences for black holes and for the cosmological constant problem which will also be the subject of our discussion in that section. Our particular interest is to look for an interface between QFT in CST and QG. Several issues which in the past were expected to delimit the interface between QFT and QG, including the two mentioned ones, are now believed to be taken care of within QFT in CST extended by backreaction.

In particular some of the estimates of the cosmological constant which are based on the filling up of energy levels, similar to the construction of the Fermi surface in condensed matter physics, are already in trouble with the QM/QFT interface. These estimates violates *local covariance* (local diffeomorphism equivalence) which as one of QFT in CST most cherished principles is basically the locality principle of QFT extended with the appropriately adapted local covariance from Einstein's classical theory. In the title of one of Hollands and Wald's papers one finds the following advice for avoiding such calculations: *Quantum Field Theory Is Not Merely Quantum Mechanics Applied to Low Energy Ef-*

fective Degrees of Freedom [18]. A model calculation without cutoff and in agreement with local covariance and backreaction can be found in [19]

2 The interface between quantum mechanics and quantum field theory

Shortly after the discovery of field quantization in the second half of the 1920s, there were two opposed viewpoints about its content and purpose represented by Dirac and Jordan. Dirac's position was that quantum theory should stand for *quantizing a true classical reality*[8] which meant field quantization for electromagnetism and particle quantization for the massive particles. Jordan, on the other hand proposed an uncompromising field quantization point of view; all what can be quantized should be quantized, independent of whether there is a classical reality or not. The more radical field quantization finally won the argument, but ironically it was Dirac's particle setting (the hole theory) and not Jordan's application of Murphy's law to all field objects which contributed the richest structural property to QFT, namely antiparticles/anticharges. It was also the hole theory in which the first perturbative QED computations (which entered the textbooks of Heitler and Wenzel) were done, before it was recognized that this setting was not really consistent. This inconsistency showed up in problems involving renormalization in which *vacuum polarization* plays the essential role. The successful perturbative renormalization of QED was also the end of hole theory and the beginning of Dirac's late conversion to QFT as the general setting for relativistic particle physics at the beginning of the 50s.

Vacuum polarization is a very peculiar phenomenon which in the special context of currents and the associated local charges of a complex free Bose field was noticed already in the 30s by Heisenberg [20]. But only when Furry and Oppenheimer [21] studied perturbative interactions of Lagrangian fields and noticed to their amazement that the Lagrangian field applied to the vacuum created inevitably some additional particle-antiparticle pairs in addition to the expected one-particle state. The number of these pairs increase with the perturbative order, pointing towards the fact that one has to deal with infinite polarization clouds in case of sharp localization. Whenever one tries in an interacting theory to create particles via local disturbances of the vacuum these vacuum polarization clouds corrupt precisely those particles which one intends to create. In the presence of interactions the notion of particles in local regions is simply meaningless; they only appear in the form of incoming and outgoing asymptotic particle configurations.

In the next subsection it will be shown that relativistic QM in the form of DPI can indeed be consistently formulated and this setting can even be extended to incorporate creation and annihilation channels [26]. This goes along way

[8] Jordan's extreme formal positivistic point of view allowed him to quantize everything which fitted into the classical Lagrangian field formalism independent of whether it had a classical reality or not.

to vindicate Dirac's relativistic particle viewpoint. But it does not vindicate it completely since theories which start as particle theories but then lead to vacuum polarization as Dirac's hole theory are at the end inconsistent. this indeed possible. Although the DPI setting will only be formulated for elastic scattering processes, it can be extended by adding creation channels

This essay does not attempt to advertise DPI as an alternative particle description to QFT, it is only meant as a conceptual challenge. By contrasting the latter with the former one learn to appreciate the conceptual depth of QFT and one becomes aware of its still unexplored regions. DPI is basically a relativistic *particle* setting i.e. it deals only with properties which can be formulated in terms of particles; this limits causality properties to macro-causality i.e. spacelike cluster factorization and timelike causal rescattering. This setting is as well understood as QM; one would be surprised to find still unilluminated regions. In contrast nobody who has studied QFT beyond a textbook level would claim to know what those postulates or axioms by which one tries to define QFT really lead to. Even now, 80 years after its discovery, one is deeply impressed that something that old can still reveal secrets. The last subsection of the present section illustrates this point by an interesting recent example.

2.1 Direct particle interactions, relativistic QM

The Coester-Polyzou theory of *direct particle interactions* (DPI), where direct means not field-mediated, is a relativistic theory in the sense of representation theory of the Poincaré group which among other things leads to a Poincaré invariant S-matrix. Every property which can be formulated in terms of particles, as the cluster factorization into systems with a lesser number of particles and other aspects of macrocausality, is fulfilled in this setting. The S-matrix does not fulfill analyticity properties as the crossing property whose derivation relies on the existence of local interpolating fields.

In contradistinction to the more fundamental locally covariant QFT, DPI is only a phenomenological setting, but one which is consistent with every property which can be expressed in terms of relativistic particles only. For a long time it was only known how to deal with *two* interacting particles. In that case one goes to the c. m. system and modifies the invariant energy operator. Assuming for simplicity identical scalar Bosons, the c.m. invariant energy operator is $2\sqrt{p^2+m^2}$ and the interaction is introduced by adding an interaction term v

$$M = 2\sqrt{\vec{p}^2+m^2} + v, \quad H = \sqrt{\vec{P}^2 + M^2} \tag{1}$$

where the invariant potential v depends on the c.m. variables p, q in an invariant manner i.e. such that M commutes with the Poincaré generators of the 2-particle system which is a tensor product of two one-particle systems.

One may follow Bakamjian and Thomas (BT) [24] and choose the Poincaré generators in their way the interaction does not affect them directly apart from the Hamiltonian. Denoting the interaction-free generators by a subscript 0 one

arrives at the following system of two-particle generators

$$\vec{K} = \frac{1}{2}(\vec{X}_0 H + H\vec{X}_0) - \vec{J} \times \vec{P}_0 (M+H)^{-1} \tag{2}$$
$$\vec{J} = \vec{J}_0 - \vec{X}_0 \times \vec{P}_0$$

The interaction v may be taken as a *local* function in the relative coordinate which is conjugate to the relative momentum p in the c.m. system; but since the scheme anyhow does not lead to local differential equations, there is not much to be gained from such a choice. The Wigner canonical spin $\vec{J}_0$ commutes with $\vec{P} = \vec{P}_0$ and $\vec{X} = \vec{X}_{.0}$ and is related to the Pauli-Lubanski vector $W_\mu = \varepsilon_{\mu\nu\kappa\lambda} P^\nu M^{\kappa\lambda}$.

As in the nonrelativistic setting, short ranged interactions v lead to Møller operators and S-matrices via a converging sequence of unitaries formed from the free and interacting Hamiltonian

$$\Omega_\pm(H, H_0) = \lim_{t \to \pm\infty} e^{iHt} e^{-H_0 t} \tag{3}$$
$$\Omega_\pm(M, M_0) = \Omega_\pm(H, H_0)$$
$$S = \Omega_+^* \Omega_-$$

The identity in the second line is the consequence of a theorem which say that the limit is not affected if instead of M we take a positive function of M as $H(M)$, as long as H_0 is the same function of M_0. This insures the *frame-independence of the Møller operators and the S-matrix.* Apart from this identity for operators and their positive functions, which is not needed in the nonrelativistic scattering, the rest behaves just as in nonrelativistic scattering theory. As in standard QM, the 2-particle cluster property is the statement that $\Omega_\pm^{(2)} \to \mathbf{1}$, $S^{(2)} \to \mathbf{1}$, i.e. the scattering formalism is identical. In particular the two particle cluster property, which says that for short range interactions the S-matrix approaches the identity if one separates the center of the wave packets of the two incoming particles, holds also for the relativistic case.

The implementation of clustering is much more delicate for 3 particles as can be seen from the fact that the first attempts were started in 1965 by Coester [22] and considerably later generalized (in collaboration with Polyzou [23]) to arbitrary high particle number. To anticipate the result below, DPI leads to a consistent scheme which fulfills cluster factorization but it has no useful second quantized formulation so it may stand accused of lack of elegance, and since we are inclined to view less elegant theories also as less fundamental, we would not trade this phenomenological relativistic theory (arbitrary potential functions instead of pontlike coupling parameters) for QFT. It is also more nonlocal and nonlinear than QM, This had to be expected since adding particles does not mean adding terms to the Hamiltonian as in Schroedinger QM.

The BT form for the generators can be achieved inductively for an arbitrary number of particles. As will be seen, the advantage of this form is that in passing from n-1 to n-particles the interactions simply add and one ends up with Poincaré group generators for an interacting n-particle system. But for $n > 2$

the aforementioned subtle problem with the cluster property arises; whereas this iterative construction in the nonrelativistic setting complies with cluster separability, this is not the case in the relativistic context.

This problem shows up for the first time in the presence of 3 particles [22]. The BT iteration from 2 to 3 particles gives the 3-particle mass operator

$$M = M_0 + V_{12} + V_{13} + V_{23} + V_{123} \tag{4}$$
$$V_{12} = M(12,3) - M_0(12;3),\ M(12,3) = \sqrt{\vec{p}_{12,3}^2 + M_{12}^2} + \sqrt{\vec{p}_{12,3}^2 + m^2}$$

and the $M(ij,k)$ result from cyclic permutations. Here $M(12,3)$ denotes the 3-particle invariant mass in case the third particle is a "spectator", which by definition does not interact with 1 and 2. The momentum in the last line is the relative momentum between the (12)-cluster and particle 3 in the joint c.m. system and M_{12} is the associated two-particle mass i.e. the invariant energy in the (12) c.m system.

As in the nonrelativistic case, one can always add a totally connected contribution. Setting this contribution to zero, the 3-particle mass operator only depends on the two-particle interaction v. But contrary to the nonrelativistic case, the BT generators constructed with M do not fulfill the cluster separability requirement as it stands. The latter demands that if the interaction between two clusters is removed, the unitary representation factorizes into that of the product of the two clusters.

One expects that shifting the third particle to infinity will render it a spectator and result in a factorization $U_{12,3} \to U_{12} \otimes U_3$. Unfortunately what really happens is that the (12) interaction also gets switched off i.e. $U_{123} \to U_1 \otimes U_2 \otimes U_3$. The reason for this violation of the cluster separability property, as a simple calculation using the transformation formula from c.m. variables to the original p_i, i = 1, 2, 3 shows [23], is that although the spatial translation in the original system (instead of the $12,3$ c.m. system) does remove the third particle to infinity as it should, unfortunately it also drives the two-particle mass operator (with which it does not commute) towards its free value which violates clustering.

In other words the BT produces a Poincaré covariant 3-particle interaction which is additive in the respective c.m. interaction terms (4), but the Poincaré representation U of the resulting system will not be cluster-separable. However, as shown first in [22], at least the 3-particle S-matrix computed in the additive BT scheme turns out to have the cluster factorization property. But without implementing the correct cluster factorization not only for the S-matrix but also for the 3-particle Poincaré generators there is no chance to proceed to a clustering 4-particle S-matrix.

Fortunately there always exist unitaries which transform BT systems into cluster-separable systems *without affecting the S-matrix.* Such transformations are called *scattering equivalences.* They were first introduced into QM by Sokolov [25] and their intuitive content is related to a certain insensitivity of

the scattering operator under quasilocal changes of the quantum mechanical description at finite times. This is reminiscent of the insensitivity of the S-matrix in QFT against local changes in the interpolating field-coordinatizations[9] by e.g. using composites instead of the Lagrangian field. The notion of scattering equivalences is conveniently described in terms of a subalgebra of *asymptotically constant operators* C defined by

$$\lim_{t\rightarrow\pm\infty} C^{\#}e^{iH_0t}\psi = 0 \tag{5}$$
$$\lim_{t\rightarrow\pm\infty}\left(V^{\#}-1\right)e^{iH_0t}\psi = 0$$

where $C^{\#}$ stands for both C and C^*. These operators, which vanish on dissipating free wave packets in configuration space, form a *-subalgebra which extends naturally to a C^*-algebra $\mathcal{C}$. A scattering equivalence is a unitary member $V\in\mathcal{C}$ which is asymptotically equal to the identity (the content of the second line). Applying this asymptotic equivalence relation to the Møller operator one obtains

$$\Omega_{\pm}(VHV^*, VH_0V^*) = V\Omega_{\pm}(H,H_0) \tag{6}$$

so that the V cancels out in the S-matrix. Scattering equivalences do however change the interacting representations of the Poincaré group according to $U(\Lambda,a)\rightarrow VU(\Lambda,a)V^*$.

The upshot is that there exists a clustering Hamiltonian H_{clu} which is unitarily related to the BT Hamiltonian H_{BT} i.e. $H_{clu} = BH_{BT}B^*$ such that $B\in\mathcal{C}$. is uniquely determined in terms of the scattering data computed from H_{BT}. It is precisely this clustering of H_{clu} which is needed for obtaining a clustering 4-particle S-matrix which is cluster-associated with the $S^{(3)}$. With the help of M_{clu} one defines a 4-particle interaction following the additive BT prescription; the subsequent scattering formalism leads to a clustering 4-particle S-matrix and again one would not be able to go to n=5 without passing from the BT to the cluster-factorizing 4-particle Poincaré group representation. Coester and Polyzou showed [23] that this procedure can be iterated and hence one arrives at the following statement

Statement: *The freedom of choosing scattering equivalences can be used to convert the Bakamijan-Thomas presentation of multi-particle Poincaré generators into a cluster-factorizing representation. In this way a cluster-factorizing S-matrix $S^{(n)}$ associated to a BT representation H_{BT} (in which clustering mass operator $M_{clu}^{(n-1)}$ was used) leads via the construction of $M_{clu}^{(n)}$ to a S-matrix $S^{(n+1)}$ which clusters in terms of all the previously determined $S^{(k)}, k<n$. The use of scattering equivalences impedes the existence of a 2^{nd} quantized formalism.*

[9] In field theoretic terminology this means changing the pointlike field by passing to another (composite) field in the same equivalence class (Borchers class) or in the setting of AQFT by picking another operator from a local operator algebra.

For a proof we refer to the original papers [23][26]. In passing we mention that the minimal extension, i.e. the one determined uniquely in terms of the two-particle interaction v) from n to n+1 for $n > 3$, contains *connected 3-and higher particle interactions* which are nonlinear expressions (involving nested roots) in terms of the original two-particle v. This is another unexpected phenomenon as compared to the nonrelativistic case.

This theorem shows that it is possible to construct a relativistic theory which only uses particle concepts only, thus correcting an old folklore which says relativity + clustering = QFT. Whether one should call this DPI theory "relativistic QM" is a matter of taste, it depends on what significance one attributes to those unusual scattering equivalences. But in any case it defines a *relativistic S-matrix setting* with the correct particle behavior. In this context one should also mention that the S-matrix bootstrap approach never addressed these macro-causality problems and this also holds for its heir the contemporary string theory.

As mentioned above Coester and Polyzou also showed that this relativistic setting can be extended to processes which maintain cluster factorization in the presence of a finite number of creation/annihilation channels, showing, as mentioned before, that the mere presence of particle creation is not characteristic for QFT[10]. Different from the nonrelativistic Schroedinger QM, the superselection rule for masses of particles which results from Galilei invariance for nonrelativistic QM does not carry over to the relativistic setting; in this respect DPI is less restrictive than its Galilei-invariant QM counterpart where such creation processes are forbidden.

Certain properties which are automatic consequences of locality in QFT but can be formulated solely in terms of particles as TCP symmetry, the existence of anti-particles, the spin-statistics connection, can be added "by hand". Other properties which are on-shell relics of locality which QFT imprints on the S-matrix and which require the notion of analytic continuation in on-shell particle momenta, as e.g. the crossing property, cannot be implemented in the QM setting of DPI.

2.2 First brush with the intricacies of the particles-field problems in QFT

Interacting QFT in contrast to QM (Schrödinger-QM or relativistic DPI), does not admit a particle interpretation at finite times[11]. If it would not be for the asymptotic scattering interpretation in terms of incoming/outgoing particles associated with the free in/out fields, there would be hardly anything of a non-fleeting nature which can be measured. In QFT in CST and thermal QFT where this particle concept is missing, the set of conceivable measurements is

[10]It does not look very likely that the S-matrix of QFT can be approximated as a limit of DPI with particle creation.

[11]Although the one-particle states and their multiparticle counterparts are global states in the Hilbert space, they are not accessible by acting locally on the vacuum. Scattering theory is the only known nonlocal intervention.

ostensibly meagre and is essentially reduced to energy- and entropy- densities as in thermal systems and black hole radiation.

Since the notion of particle is often used in a more general sense than in this paper, it may be helpful to have an interlude on this topic. By particle I mean an asymptotically stable object which forms the tensor product basis for an asymptotically complete description. It is precisely the particle concept which furnishes QFT with a (LSZ, Haag-Ruelle) complete asymptotic particle interpretation[12], so that a Fock space tensor structure is imposed on the Hilbert space of the interacting system. The physics behind it is the idea that if we were to cobble the asymptotic spacetime region with counters and monitor coincidences of localization events, then the n-fold coincidence/anticoincidence (the latter in order to insure that we caught all particles) set up would eventually remain stable because the far removed localization centers would have ceased to interact and from there on move freely i.e. one would be in a region where the Newton-Wigner adaptation of the Born position operator would lead to genuinely Poincaré invariant transition probabilities.

The particle concept in QFT is therefore precisely applicable where it is needed, namely for asymptotically separated Born-localized events; thus the invariant S-matrix has no memory about the reference-system-dependent Born localization and the question of what particle counters really count in finite regions becomes academic. In fact the careless use of the B-N-W localization for finite distances is known to lead to unphysical superluminal effects; in that case one should formulate the problem in the setting of the modular localization.

Tying up the particle concept in QFT to asymptotically stable counter-coincidences can be traced back to a seminal paper by Haag and Swieca [27]. In that paper it was noticed for the first time that the phase space volume in QFT unlike that in QM is not finite but its cardinality is very mild (the phase space is *nuclear*). This is yet another line of unexpected different consequences [28] resulting from the different localization concepts in QM and QFT, but this interested topic will not be pursued here.

Not all particles comply with this definition; in fact all electrically charged particles are *infraparticles* i.e. objects which are asymptotically stable up to an unobserved cloud of infinitely may infrared photons whose presence has the consequence that instead of the mass shell $p^2 = m^2$ the mass m of the charged particle only denotes the start of cut instead of a mass shell delta function [29]. The naive scattering theory leads to infrared divergencies which cannot be cured by renormalizing parameters, but rather requires a significant change of scattering theory. Since for the problems at hand this is of less importance we leave it at these remarks.

It is the *asymptotic particle structure* which leads to the observational richness of QFT. Once we leave this setting by going to curved spacetime or to QFT in KMS thermal states representations, or if we restrict a Minkowski spacetime theory to a Rindler wedge with the Hamiltonian being now the boost operator

[12] The asymptotic completeness property was for the first time established (together with a recent existence proof) in a family of factorizing two-dimensional models (see the section on modular localization) with nontrivial scattering.

with its two-sided spectrum, we are loosing the setting of scattering theory of particles and its observational wealth. The restriction to the Rindler world preserves the Fock space particle structure of the free field Minkowski QFT, but it looses its direct intrinsic significance with respect to the Rindler situation[13]. In the Rindler world since the Minkowski vacuum is now a thermal state and there is no particle scattering theory in the "boost time" in such a thermal situation.

Of course there remains the possibility to measure thermal excitations in an *Unruh counter* [30] or to use a counter to register Hawking [31] radiation. or to determine the energy density in a cosmological reference state (see also last section). In this case one is not measuring individual particles but rather an energy density. An famous example for a kind of measurement with great physical significance for the development of cosmology is the cosmic background radiation which is the expectation of the energy density in the cosmic reference state. In such situations one does not only loose the Poincaré symmetry but together with it the vacuum as well as the particle state.

This raises the question whether the result of over 60 years accumulated knowledge about scattering of particles still find a conceptual place in the more general QFT in CST or whether the only data consistent CST are those obtained placing a counters into a cosmic reference state and measuring expectations of the energy. On one extreme end are those who claim that particles have no place in QFT in CST, one is rather forced to abandon particles altogether and adopt the point that one measures fields, in particular the energy density in the cosmic state. This point of view one finds in particular in recent publications of Wald [34]. The only kind of field for which one can envisage a field measurements without thinking in terms of particles is the electromagnetic field apart from this exception all other fields serve a interpolating fields for particles. So the question what fields interpolate in the setting of CST, if not particles, remains open.

Formally the local covariance principle forces the construction of a QFT on all causally complete manifolds and their submanifolds at once. So the QFT in Minkowski spacetime with its particle interpretation is always part of the solution. What one would like to have is a more direct physical connection e.g. a particle concept in the tangent space or something in this direction.

The conceptual differences between a DPI relativistic QM and QFT are enormous, but in order to appreciate this, one has to become acquainted with structural properties of QFT which are somewhat removed from the standard properties of the Lagrangian setting and therefore have not entered textbooks; it is the main purpose of the following sections to highlight these contrasts by going more deeply into QFT.

There are certain folkloric statements about the relation QM–QFT whose

[13] There is of course the mathematical possibility of choosing a groundstate representation for a Rindler world instead of restricting the Minkowski vacuum. In that case it is not clear whether in the presence of interactions the exitations above this ground state have the Haag-Swieca asymptotic localization stability i.e. whether scattering theory applies to such a situation. It would be interesting to (dis)prove the validity of Haag-Ruelle scattering theory in such a situation.

dismissal does not require much conceptual sophistication. For example in trying to make QFT more susceptive to newcomers it is sometimes said that a free field is nothing more than a collection of infinitely many coupled oscillators. Although not outright wrong, this characterization misses the most important property of how spacetime enters as an ordering principle into QFT. It would not help any newcomer who knows the quantum oscillator, but has not met a free field before, to construct a free field from such a verbal description. Even if he manages to write down the formula of the free field he would still have to appreciate that the most important aspect is the causal localization and not that what oscillates. This is somewhat reminiscent of the alleged virtue from equating QM via Schrödinger's formulation with classical wave theory. What may be gained for a newcomer by appealing to his computational abilities acquired in classical electrodynamics, is more than lost in the conceptual problems which he confronts later when facing the subtleties of quantum physics.

2.3 Quantum mechanical Born localization versus covariant localization in LQP

Let us know come to the main point namely the difference between QM and LQP in terms of their localization concept. As it should be clear from previous remarks we will use the word *Born localization* for the probability density of the x-space Schroedinger wave function $p(x) = |\psi(x)|^2$; the adaptation to the invariant inner product of relativistic wave functions was done by Newton and Wigner [32] and will be continued to be referred to as B-N-W localization. Being a bona fide probability density, one may characterize the Born localization in a spatial region $R \in \mathbb{R}^3$ at a given time in terms of a localization projector $P(R)$. The standard version of QM and the various settings of measurement theory rely heavily on these projectors. Without Born localization and the ensuing projectors it would be impossible to formulate the conceptual basis for the time-dependent scattering theory.

The B-N-W position operator and its family of spatial region-dependent projectors are not covariant under Lorentz boosts. For Wigner, who was not aware of the existence of the covariant modular localization, this frame dependence raised doubts about the conceptual soundness of QFT. Apparently the existence of completely covariant correlation functions in renormalized perturbation theory did not satisfy him, he wanted an understanding from first principles.

The fact that modular localization remained closed to Wigner may be seen as an indication of its subtlety; the standard operator formalism as used in QM contains not the slightest hint in its direction.

The lack of covariance of B-N-W localization in finite time propagation leads to frame-dependence and superluminal contributions, which is why the terminology "relativistic QM" has to be taken with a grain of salt. However, as already emphasized, in the asymptotic limit of large timelike separation as required in scattering theory, the covariance, frame-independence and causal relations are recovered. With other words one obtains a Poincaré-invariant unitary S-matrix whose DPI construction can also be shown to guaranty also the validity of all the

macro-causality requirements (spacelike clustering, absence of timelike precursors, causal rescattering) which can be formulated in a particle setting without taking recourse to interpolating local fields. Even though the localizations of the individual particles are frame-dependent, the asymptotic relation between N-W-localized events is given in terms of the geometrically associated *covariant on-shell momenta* or 4-velocities. In fact all observations on particles always involve B-N-W localization measurements.

The situation of propagation of DPI is similar to that of propagation of acoustic waves in an elastic medium; although in neither case there is a limiting velocity there exists a maximal "effective" velocity, for DPI this is c and in the acoustic case this is the velocity of sound.

In comparing QM with QFT it is often convenient in discussions about conceptual issues to rephrase the content of (nonrelativistic) QM in terms of operator algebras and states in the sense of positive expectation functional on operator algebras; in this way one also achieves more similarity with the formalism of QFT where this abstraction becomes important. In this Fock space setting the basic operators are creation/annihilation operators $a^{\#}(\mathbf{x})$ with

$$[a(\mathbf{x}), a^{*}(\mathbf{y})]_{grad} = \delta(\mathbf{x} - \mathbf{y}) \tag{7}$$

where for Fermions the graded commutator is the anticommutator. The ground state for T=0 zero matter density states is annihilated by $a(\mathbf{x})$ whereas for fine density one has a filled below the Fermi surface state for Fermions and a Bose-Einstein condensate for Bosons. In QFT the identification of pure states with state-vectors of a Hilbert space has no intrinsic meaning and often cannot be maintained in concrete situations. For the same reasons of achieving a unified description we use the multi-particle (Fock space) setting instead of the Schroedinger formulation. Although DPI is formulated in Fock space there is no second quantized formalism (7), which renders the formalism less elegant and more detailed than its nonrelativistic counterpart.

The global algebra which contains all observables independent of their localization is the algebra $B(H)$ of all bounded operators in Hilbert space. Physically important unbounded operators are not members but rather have the mathematical status of being affiliated with $B(H)$ and its subalgebras; this bookkeeping makes it possible to apply powerful theorems from the theory of operator algebras (whereas unbounded operators are treated on a case to case basis). $B(H)$ is the correct global description whenever the physical system under discussion arises as the weak closure of a ground state representation of an irreducible system of operators[14] be it QM or LQP. According to the classification of operator algebras, $B(H)$ and all its multiples are of Murray von Neumann type I_{∞} whose characteristic property is the existence of minimal projectors; in the irreducible case these are the one-dimensional projectors belonging to measurements which cannot be refined. There are prominent physical

[14]The closure in a thermal equilibrium state associated with a continuous spectrum Hamiltonian leads to a unitarily inequivalent (type III) operator algebra without minimal projectors.

states which lead to different global situations as e.g. thermal KMS states but for the time being our interest is in ground states.

The structural differences between QM and LQP emerge as soon as one uses localization in order to provide a physical substructure to $B(H)$. It is well known that a dissection of space into nonoverlapping spatial regions i.e. $\mathbb{R}^3 = \cup_i R_i$ implies via Born localization a tensor factorization of $B(H)$ and H

$$\begin{aligned} B(H) &= \bigotimes_i B(H(R_i)) && (8) \\ H &= \bigotimes_i H(R_i),\ P(R_i)H = H(R_i) \\ \tilde{\mathbf{X}}_{op} &= \int a^*(\vec{x})\vec{x}a(\vec{x})d^3x = \int \vec{x}dP(\vec{x}) && (9) \end{aligned}$$

where the third line contains the definition of the position operator and its spectral decomposition in the bosonic Fock space. Hence there is orthogonality between subspaces belonging to localizations in nonoverlapping regions (orthogonal Born projectors) and one may talk about states which are pure in $H(R_i)$. A pure state in the global algebra $B(H)$ may not be of the tensor product form but may rather describe a superposition of factorizing states; the Schmidt decomposition is a method to achieve this with an intrinsically determined basis in the two factors. States which are not tensor products but rather superpositions of such are called entangled and their reduced density matrix obtained by averaging outside a region R_i describes a mixed state on $B(H(R_i))$. This is the standard formulation of QM in which pure states are vectors and mixed states are density matrices.

Although this quantum mechanical entanglement can be related with the notion of entropy, it is an entropy in the sense of *information theory* and not in the thermal sense of thermodynamics, i.e. one cannot create a temperature as a quantitative measure of the degree of quantum mechanical entanglement which results from Born-restricting pure global states to a finite region and its outside environment. The net structure of $B(H)$ in terms of the $B(H(R_i))$ is of a kinematical kind, it does not create a new Hamiltonian with respect to which the reduced state becomes a KMS state. The quantum mechanical dynamics through a Hamiltonian shows that the tensor factorization from Born localization at one time is almost instantaneously lost in the time-development, as expected of a theory of without a maximal propagation speed.

The LQP counterpart of the Born-localized subalgebras at a fixed time are the observable algebras $\mathcal{A}(\mathcal{O})$ for causally completed ($\mathcal{O} = \mathcal{O}''$, the causal complement taken twice) spacetime regions $\mathcal{O}$; they form what is called in the terminology of LQP a *local net* $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O}\subset M}$ of operator algebras indexed by regions in Minkowski spacetime $\cup\mathcal{O} = \mathcal{M}$ which is subject to the natural and obvious requirements of isotony ($\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ if $O_1 \subset O_2$) and causal locality, i.e. the algebras commute for spacelike separated regions.

The connection with the standard formulation of QFT in terms of pointlike fields is that smeared fields $\Phi(f) = \int \Phi(x)f(x)d^4x$ with $suppf \subset \mathcal{O}$ under

reasonable general conditions generate local algebras. Pointlike fields, which by themselves are too singular to be operators (even if admitting unboundedness), have a well-defined mathematical meaning as operator-valued distributions. But as mentioned before, there are myriads of fields which generate the same net of local operator algebras, hence they play a similar role in LQP as coordinates in modern differential geometry i.e. they coordinatize the net of spacetime indexed operator algebras and only the latter has an intrinsic meaning. But as the use of particular coordinates often facilitates geometrical calculations, the use of particular fields with e.g. the lowest short-distance dimension within the infinite charge equivalence class of fields can greatly simplify algebraic calculations in QFT. Therefore it is a problem of practical importance to construct a covariant basis of locally covariant pointlike fields of an equivalence class.

For massive free fields and for massless free fields of finite helicity such a basis is especially simple; the "Wick-basis" of composite fields still follows in part the logic of classical composites. This remains so even in the presence of interactions in which case the Wick-ordering gets replaced by the technically more demanding "normal ordering". For free fields in CST and the definition of their composites it is important to require the *local covariant transformation behavior* under local isometries [33]. The conceptual framework in the presence of interactions has also been understood [35].

We now return to the main question namely what changes if we pass from the Born localization of QM to the causal localization of LQP? The crucial property is that a localized algebra $\mathcal{A}(\mathcal{O}) \subset B(H)$ together with its commutant $\mathcal{A}(\mathcal{O})'$ (which under very general conditions[15] is equal to algebra of the causal disjoint of $\mathcal{O}$ i.e. $\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}'))$ are two von Neumann factor algebras i.e.

$$B(H) = \mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O})',\ \mathcal{A}(\mathcal{O}) \cap \mathcal{A}(\mathcal{O})' = \mathbb{C}\mathbf{1} \tag{10}$$

In contrast to the QM algebras the local factor algebras are not of type I and $B(H)$ does *not tensor-factorize* in terms of them, in fact they cannot even be embedded into a $B(H_1) \otimes B(H_2)$ tensor product. The prize to pay for ignoring this important fact and imposing wrong structures is the appearance of spurious ultraviolet divergences. On the positive side, as will be seen later, without this significant change in the nature of algebras there would be no holography onto causal horizons, no thermal behavior caused by localization and a fortiori no area-proportional localization entropy.

In QM a pure state vector, which, with respect to a distinguished tensor product basis in $H(R) \otimes H(R \backslash \mathbb{R}^3)$, is a nontrivial superposition of tensor-basis states, will be generally become impure state if restricted to $B(H(R))$; in the standard formalism (where only pure states are represented by vectors) it is described by a density matrix. This phenomenon of *entanglement* is best described by the *information theoretic notion of entropy.* On the other hand each pure state on $B(H(R))$ or $B(H(R \backslash \mathbb{R}^3))$ originates from a pure state on $B(H)$.

[15] In fact this duality relation can always be achieved by a process of maximalization (Haag dualization) which increases the degrees of freedom inside $\mathcal{O}$. A pedagogical illustration based on the "generalized free field" can be found in [36].

The situation in LQP is radically different since the local algebras as $\mathcal{A}(\mathcal{O})$ have *no pure states at all*; so the dichotomy between pure and mixed states breaks down and the kind of entanglement caused by field theoretic localization is much more violent then that coming from Born-localization[16] (see below). Unlike Born localization, causal localization is not related to position operators and projectors $P(R)$; rather the operator algebras $\mathcal{A}(\mathcal{O})$ are of an entirely different kind than those met in ground state QM (zero temperature); they are all isomorphic to one abstract object, the hyperfinite type III_1 von Neumann factor also referred to as *the monad.* As will be seen later LQP creates its wealth from just this one kind of brick; all the structural richness comes from positioning the bricks, there is nothing in the bricks themselves. In a later section it will later be explained how this emerges from modular localization and a related operator formalism.

The situation does not change if one takes for $\mathcal{O}$ a region R at a fixed time; in fact in a theory with finite propagation one has$\mathcal{A}(R) = \mathcal{A}(D(R))$ where $D(R)$ is the diamond shaped double cone subtended by R (the causal shadow of R). Even if there are no pointlike generators and if the theory only admits a macroscopically localized net of algebras (e.g. a net of non-trivial wedge-localized factor algebras $\mathcal{A}(W)$ which leads to trivial double cone intersection algebras $\mathcal{A}(\mathcal{O})$), the algebras would still not tensor factorize i.e. $B(H) \neq \mathcal{A}(W) \otimes \mathcal{A}(W')$. Hence the properties under discussion are not directly related to the presence of singular generating fields but are connected to the existence of well-defined causal shadows. It turns out that there is a hidden singular aspect in the sharpness of the $\mathcal{O}$-localization which generates infinitely large vacuum polarization clouds on the causal horizon of the localization. In a later section a method (splitting) will be presented which permits to define a split-distance dependent but otherwise intrinsically defined finite thermal entropy.

Many divergencies in QFT are the result of conceptual errors in the formulation resulting from tacitly identifying QFT with some sort of relativistic QM[17], especially computations which ignore the singular nature of pointlike localized fields. Conceptual mistakes are facilitated by the fact that even nonlocal but covariant objects are singular; this is evident from the Kallen-Lehmann representation of a covariant scalar object

$$\langle A(x)A(y)\rangle = \int \Delta_+(x-y,\kappa^2)\rho(\kappa^2)d\kappa^2 \tag{11}$$

which was proposed precisely to show that even without demanding locality, but retaining only covariance and the Hilbert space structure (positivity), a certain singular behavior of covariant objects is unavoidable. In the DPI scheme this was avoided because covariance there is limited to asymptotic relations.

[16] By introducing in addtion to free fields $A(x)$ which are covariant Fourier transforms also noncovariant Fourier transforms $a(\vec{x},t), a^*(\vec{x},t)$ one can explicitly that they are relatively nonlocal.

[17] The correct treatment of perturbation theory which takes into account the singular nature of pointlike quantum fields may yield more free parameters than in the classical setting, but one is never required to confront infinities or cut-offs.

In the algebraic formulation the covariance requirement refers to the geometry of the localization region $\mathcal{A}(\mathcal{O})$ i.e.

$$U(a,\Lambda)\mathcal{A}(\mathcal{O})U(a,\Lambda)^{*}=\mathcal{A}(\mathcal{O}_{a,\Lambda}) \tag{12}$$

whereas no additional requirement about the transformation behavior under finite (tensor, spinor) Lorentz representations (which would bring back the unboundedness and thus prevent the use of powerful theorems in operator algebras) is imposed for the individual operators. The singular nature of pointlike generators (if they exist) is then a purely mathematical consequence. Using such singular objects in pointlike interactions in the same way as one uses operators in QM leads to self-inflicted divergence problems however the divergence problems for zero splitting distance caused by vacuum fluctuations near a causal or event horizon are genuine and may very well be the only true divergence problems in LQP.

We have seen that although QM and QFT can be described under a common mathematical roof (C^{*}-algebras with a state functional), as soon as one introduces the physically important localization structure, significant conceptual differences appear. These differences are due to the presence of vacuum polarization in QFT as a result of causal localization and they tend to have dramatic consequences; the most prominent ones will be presented in this and the subsequent sections, as well as in the second part.

The net structure of the observables allows a local comparison of states: two states are locally equal in a region $\mathcal{O}$ if and only if the expectation values of all operators in $\mathcal{A}(\mathcal{O})$ are the same in both states. Local deviations from any state, in particular from the vacuum state, can be measured in this manner, and states that are indistinguishable from the vacuum in the causal complement of some region ('strictly localized states' [37]) can be defined. Due to the unavoidable correlations in the vacuum state in relativistic quantum theory (the Reeh-Schlieder property [3]), the space $H(\mathcal{O})$ obtained by applying the operators in $\mathcal{A}(\mathcal{O})$ to the vacuum is, for any open region $\mathcal{O}$, dense in the Hilbert space and thus far from being orthogonal to $H(\mathcal{O}')$. This somewhat counter-intuitive fact is inseparably linked with a structural difference between the local algebras and the algebras encountered in non-relativistic quantum mechanics (or the global algebra of a quantum field associated with the entire Minkowski space-time) as mentioned in connection with the breakdown of tensor-factorization (10).

The result is a particular benevolent form of "Murphy's law" for interacting QFT: *everything which is not forbidden (by superselection rules) to couple, really does couple*. On the level of interacting particles this has been termed *nuclear democracy*: Any particle whose superselected charge is contained in the spectrum resulting from fusing charges in a cluster of particles can be viewed as a bound state of that cluster. This renders interacting QFT conceptually much more attractive and fundamental than QM, but it also contributes to its computational complexity if one tries to access it using operator or functional methods from QM. The latter method also are responsible for the occurance of those infinities in the first place which one then "renormalizes" away by invoking the

distinction between Lagrangian and physical coupling and mass parameters. If one does perturbative QFT according to its own principles there is never any infinity, but the recursive implementation of the principles may generate paramters which were not on ones mind at the beginning (the Epstein-Glaser iteration).

It is believed that any violation of of the above Murphy's law also violates the setting of pointlike generated QFT. So the only known approach to particle physics which is not subject to this law and at least maintains macro-causality is the before presented quantum mechanical DPI setting[18]. Whereas the latter has minimal projections corresponding to optimal observations, this is not so for the local algebras which turn out to be of type III (in the terminology of Murray and von Neumann); in these algebras every projection is isometrically equivalent to the largest projector which is the identity operator. Some physical consequences of this difference have been reviewed in [38]. The claim is not that subalgebras of QM cannot be of type III but rather that physical subalgebras obtained by the operator methods of QM (in particular by Born localization) remain of type I.

The Reeh-Schlieder property [3] (in more popular but less precise terminology: the state-field relation) also implies that the expectation value of a projection operator localized in a bounded region *cannot* be interpreted as the probability of detecting a particle-like object in that region, since it is necessarily nonzero if acting on the vacuum state. Our later study reveals that the restriction of the vacuum (or any other global finite energy state) to $\mathcal{A}(\mathcal{O})$ is entangled in a much more radical sense than the ground state of a quantum mechanical system under the spatial inside/outside split. The reduced ground state on $A(\mathcal{O})$ transmutes into a KMS thermal state at a appropriately normalized (Hawking) temperature[19]. The intrinsically defined *modular "Hamiltonian"* associated via modular operator theory to standard pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ is always available in the mathematical sense but allows a physical interpretation only in those rare cases when it coincides with one of the global spacetime generators. Well known cases are. the Lorentz boost for the wedge region in Minkowski spacetime (the Unruh effect) and the positive generator of a double-cone preserving conformal transformation in a conformal theory. This phenomenon has the same origin as the later discussed universal area proportionality of localization entropy which is the entropic side of modular localization).

There exists in fact a whole family *of modular Hamiltonians* since the operators in $\mathcal{A}(\mathcal{O})$ naturally fulfill the KMS condition of any standard pair $(\mathcal{A}(\check{\mathcal{O}}), \Omega_{vac})$ for $\check{\mathcal{O}} \supset \mathcal{O}$: how the different modular thermal states physically "out themselves" depends on which larger system one wants the operators in $\mathcal{A}(\mathcal{O})$ to be associated with, i.e. it depends on the $\check{\mathcal{O}}$-localization of the observers. The

[18] It is quite probable that semiinfinite stringlike interactions violate Murphy's law as well as the crossing property. But even then they would be still valid in the subalgebra/subspace of local observables.

[19] The effects we are concerned with are ridiculously small and probably never mearurable, but here we are interested in principle aspects of the most successfull and fundamental theory and not in FAPP issues.

original system has no preference for a particular modular Hamiltonian, it fulfills all those different KMS properties with respect to all those infinitely many different modular Hamiltonians $H_{\mathrm{mod}}(\mathcal{O})$ simultaneously. In certain cases there is a preferred region where this situation of extreme virtuality caused by vacuum polarization passes to real physics. The most interesting and prominent case comes about when spacetime curvature is creating a black hole[20]. In such a situation the fleeting "as if" aspect of a causal localization horizon (e.g. the Unruh horizon) changes to give room for a more real *event horizon.* For computations of thermal properties however, including thermal entropy, it does not matter whether the horizon is a fleeting causal localization horizon or a "real" curvature generated black hole event horizon. This leads to a picture about the LQP-QG (quantum gravity) interface which is somewhat different from that in most of the literature; we will return to these issues in connection with the presentation of the *split property* in the section on algebraic modular aspects.

A direct comparison with B-N-W-localization can be made in the case of free fields which are well defined as operator valued distributions in the space variables at a fixed time. The one-particle states that are B–N-W localized in a given space region at a fixed time are not the same as the states obtained by applying field operators smeared with test functions supported in this region to the vacuum. The difference lies in the non-local energy factor $\sqrt{p^2+m^2}$ linking the non-covariant B-N-W states with the states defined in terms of the covariant field operators.

Causality in relativistic quantum field theory is mathematically expressed through local commutativity, i.e., mutual commutativity of the algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O}')$. There is an intimate connection of this property with the possibility of preparing states that exhibit no mutual correlations for a given pair of causally disjoint regions. In fact, in a recent paper Buchholz and Summers [5] show that local commutativity is a necessary condition for the existence of such uncorrelated states.

Conversely, in combination with some further properties (split property [40], existence of scaling limits) that are physically plausible, have been verified in models and follow from assumption about what constitutes a physical phase space degree of freedom cardinality in QFT, local commutativity leads to a very satisfactory picture of statistical independence and local preparabilty of states in relativistic quantum field theory. We refer to [41][42] for thorough discussions of these matters and [38][10] for a brief review of some physical consequences. The last two papers explain how the above mentioned concepts avoids the defects of the NW localization and resolve spurious problems rooted in assumptions that are in conflict with basic principles of relativistic quantum physics. In particular it can be shown how an alleged difficulty [6][7] with Fermi's famous Gedankenexperiment, which Fermi proposed in order to show that the velocity of light is also the limiting propagation velocity in quantum electrodynamics, can be resolved by taking [38] into account the progress on the

[20]Even in that case there is no difference whether one associates the localization property with the outside, inside, or with the horizon of the black hole.

conceptual issues of causal localization and the gain in mathematical rigor since the times of Fermi.

After having discussed some significant conceptual differences between QM and LQP, one naturally asks for an argument why and in which way QM appears as a nonrelativistic limit of LQP. The standard kinematical reasoning of the textbooks is acceptable for fermionic/bosonic systems in the sense of FAPP, but has not much strength on the conceptual level. To see its weakness, imagine for a moment that we would live in a 3-dim. world of anyons (abelian plektons, where plektons are Wigner particles with braid group statistics). Such relativistic objects are by their very statistics so tightly interwoven that there simply are no compactly localized free fields which only create a localized anyon without a vacuum polarization cloud admixture. In such a world no nonrelativistic limit which maintains the spin-statistic connection could lead to QM, the limiting theory would rather *remain a nonrelativistic QFT*. There is simply no Schrödinger equation for plektonic particle-like objects which carry the spin/statistics properties of anyons. In 4-dimensional spacetime there is no such obstacle against QM, simply because there exist relativistic free fields whose application to the vacuum generates a vacuum-polarization-free one-particle state and the spin-statistics structure does not require the permanence of polarization clouds in the nonrelativistic limit.

2.4 Modular localization

Previously it was mentioned on several occasions that the localization underlying QFT can be freed from the contingencies of field coordinatizations. This is achieved by a physically as well mathematically impressive but for historic and sociological reasons little known theory. Its name "modular theory" is of mathematical origin and refers to a substantial generalization of the (uni)modularity encountered in the relation between left/right Haar measure in group representation theory. In the middle 60s the mathematician Tomita presented a vast generalization of this theory to operator algebras and in the subsequent years this theory received essential improvements from Takesaki.

At the same time the physicists Haag, Hugenholtz and Winnink published their work on statistics mechanics of open systems [3]. When the physicists and mathematicians met at a conference in Baton Rouge in 1966, there was surprise and satisfaction about the perfection with which these independent developments supported each other. Physicists not only adapted mathematical terminology, but mathematicians also took some of their names from physicists as e.g. KMS states which refer to Kubo, Martin and Schwinger who introduced an analytic property of Gibbs states merely as a computational tool (in order to avoid computing traces). Haag Hugenholtz and Winnink realized that this property (which they termed the KMS property) is the only property which survives in the thermodynamic limit when the trace formulas becomes divergent.

This turned out to be the right concept for formulating and solving problems directly in the setting of open systems. In the present work the terminology is mainly used for thermal states which are not Gibbs density. They are typical

for LQP for example every multiparticle state $\Omega_{particle}$ of finite energy including the vacuum (i.e. every physical particle state) upon restriction to a local algebra $\mathcal{A}(\mathcal{O})$ becomes a KMS state with respect to a "modular Hamiltonian" which is canonically determined by $(\mathcal{A}(\mathcal{O}), \Omega_{particle})$.

Connes, in his path-breaking work on the classification of von Neumann factors, made full use of this hybrid math-phys. terminology. Nowadays one can meet mathematicians who use the KMS property but do not know its origin. One can hardly think of any other confluence of mathematical and physical ideas on such a profound and at the same time natural level, even including the beginnings of QT when the mathematical apparatus needed for QM already existed.

About 10 years later Bisognano and Wichmann [14] discovered that a vacuum state restricted to a wedge-localized operator algebra $\mathcal{A}(W)$ in QFT defines a modular setting in which the restricted vacuum becomes a thermal KMS state with respect to the wedge-affiliated L-boost "Hamiltonian". This step marks the beginning of a very natural yet unexpected relation between thermal and geometric properties which is totally characteristic for QFT which is not shared by classical theory nor by QM. Thermal aspects of black holes were however discovered independent of this work, and the first physicist who saw the connection was Geoffrey Sewell [15].

The theory becomes more accessible for physicists if one introduces it first in its more limited spatial- instead of its full algebraic- context. Since as a foundational structure of LQP it merits more attention than it hitherto received from the particle physics community, I will present some of its methods and achievements.

It has been realized by Brunetti, Guido and Longo [9] [21] there is a natural localization structure on the Wigner representation space for any positive energy representation of the proper Poincaré group. Upon second quantization this representation theoretical determined localization theory gives rise to a local net of operator algebras on the Wigner-Fock space over the Wigner representation space.

The starting point is an irreducible representation $U_1$of the Poincaré´group on a Hilbert space H_1 that after "second quantization" becomes the single-particle subspace of the Hilbert space (Wigner-Fock-space) H_{WF} of the field[22]. The construction then proceeds according to the following steps [9][43][10]. To maintain simplicity we limit our presentation to the bosonic situation.

One first fixes a reference wedge region, e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > \left|x^0\right|\}$ and considers the one-parametric L-boost group (the hyperbolic rotation by χ in the $x^{d-1} - x^0$ plane) which leaves W_0 invariant; one also needs the reflection j_{W_0} across the edge of the wedge (i.e. along the coordinates $x^{d-1} - x^0$). The Wigner representation is then used to define two commuting wedge-affiliated operators

$$\delta_{W_0}^{it} = \mathfrak{u}(0, \Lambda_{W_0}(\chi = -2\pi t)),\ \mathfrak{j}_{W_0} = \mathfrak{u}(0, j_{W_0}) \tag{13}$$

[21] With somewhat different motivations and lesser mathematical rigor also in [11].

[22] The construction works for arbitrary positive energy representations, not only irreducible ones.

where attention should be paid to the fact that in a positive energy representation any operator which inverts time is necessarily antilinear[23]. A unitary one-parametric strongly continuous subgroup as $\delta_{W_0}^{it}$ can be written in terms of a selfadjoint generator as $\delta_{W_0}^{it} = e^{-itK_{W_0}}$ and therefore permits an "analytic continuation" in t to an unbounded densely defined positive operators $\delta_{W_0}^{s}$. With the help of this operator one defines the unbounded antilinear operator which has the same dense domain.

$$\mathfrak{s}_{W_0} = \mathfrak{j}_{W_0}\delta_{W_0}^{\frac{1}{2}} \tag{14}$$
$$\mathfrak{j}\delta^{\frac{1}{2}}\mathfrak{j} = \delta^{-\frac{1}{2}} \tag{15}$$

Whereas the unitary operator $\delta_{W_0}^{it}$ commutes with the reflection, the antiunitarity of the reflection changes causes a change of sign in the analytic continuation as written in the second line. This leads to the idempotency of the s-operator on its domain as well as the identity of its range with its domain

$$\mathfrak{s}_{W_0}^2 \subset \mathbf{1}$$
$$dom\ \mathfrak{s} = ran\ \mathfrak{s}$$

Such operators which are unbounded and yet involutive on their domain are very unusual; according to my best knowledge they only appear in modular theory and it is precisely these unusual properties which are capable to encode geometric localization properties into domain properties of abstract quantum operators. The more general algebraic context in which Tomita discovered modular theory will be mentioned later.

The idempotency means that the s-operator has ± 1 eigenspaces; since it is antilinear the +space multiplied with i changes the sign and becomes the -space; hence it suffices to introduce a notation for just one eigenspace

$$\begin{aligned} \mathfrak{K}(W_0) &= \{domain\ of\ \Delta_{W_0}^{\frac{1}{2}},\ \mathfrak{s}_{W_0}\psi = \psi\} \qquad (16) \\ \mathfrak{j}_{W_0}\mathfrak{K}(W_0) &= \mathfrak{K}(W_0') = \mathfrak{K}(W_0)',\ duality \\ \overline{\mathfrak{K}(W_0) + i\mathfrak{K}(W_0)} &= H_1,\ \mathfrak{K}(W_0) \cap i\mathfrak{K}(W_0) = 0 \end{aligned}$$

It is important to be aware that, unlike QM, we are here dealing with real (closed) subspaces $\mathfrak{K}$ of the complex one-particle Wigner representation space H_1. An alternative which avoids the use of real subspaces is to directly deal with complex dense subspaces as in the third line. Introducing the graph norm of the dense space the complex subspace in the third line becomes a Hilbert space in its own right. The second and third line require some explanation. The upper dash on regions denotes the causal disjoint (which is the opposite wedge) whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on H_1.

[23]The wedge reflection $\mathfrak{j}_{W_0}$ differs from the TCP operator only by a π-rotation around the W_0 axis.

The two properties in the third line are the defining property of what is called the *standardness property* of a real subspace[24]; any standard K space permits to define an abstract s-operator

$$\mathfrak{s}(\psi + i\varphi) = \psi - i\varphi \tag{17}$$
$$\mathfrak{s} = \mathfrak{j}\delta^{\frac{1}{2}}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group δ^{it} and a antiunitary reflection which generally have however no geometric significance. The domain of the Tomita s-operator is the same as the domain of $\delta^{\frac{1}{2}}$ namely the real sum of the K space and its imaginary multiple. Note that this domain is determined by Wigner group representation theory only.

It is easy to obtain a net of K-spaces by $U(a,\Lambda)$-transforming the K-space for the distinguished W_0. A bit more tricky is the construction of sharper localized subspaces via intersections

$$\mathfrak{K}(\mathcal{O}) = \bigcap_{W \supset \mathcal{O}} \mathfrak{K}(W) \tag{18}$$

where $\mathcal{O}$ denotes a causally complete smaller region (noncompact spacelike cone, compact double cone). Intersection may not be standard, in fact they may be zero in which case the theory allows localization in W (it always does) but not in $\mathcal{O}$. Such a theory is still causal but not local in the sense that its associated free fields are pointlike.

There are three classes of irreducible positive energy representation, the family of massive representations $(m > 0, s)$ with half-integer spin s and the family of massless representation which consists really of two subfamilies with quite different properties namely the $(0, h)$, h half-integer class (often called the neutrino, photon class), and the rather large class of $(0, \kappa > 0)$ infinite helicity representations parametrized by a continuous-valued Casimir invariant κ [10].

For the first two classes the $\mathfrak{K}$-space is standard for arbitrarily small $\mathcal{O}$ but this is definitely not the case for the infinite helicity family for which the compact localization spaces turn out to be trivial[25]. Their tightest localization, which still permits nontrivial (in fact standard) $\mathfrak{K}$-spaces for *all* positive energy representations, is that of a *spacelike cone* with an arbitrary small opening angle whose core is a *semiinfinite string* [9]; after "second quantization (see next subsection) these strings become the localization region of string-like localized covariant generating fields[26]. The modular localization of states, which is gov-

[24]According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

[25]It is quite easy to prove the standardness for spacelike cone localization (leading to singular stringlike generating fields) just from the positive energy property which is shared by all three families [9].

[26]The epithet "generating" refers to the tightest localized singular field (operator-valued distribution) which generates the spacetime-indexed net of algebras in a QFT. In the case of localization of states the generators are state-valued distributions.

erned by the unitary representation theory of the Poincaré group, has only two kind of generators: pointlike state and semiinfinite stringlike states; generating states of higher dimensionality (brane states) can be excluded.

Although the observation that the third Wigner representation class is not pointlike generated was made many decades ago, the statement that it is semi-infinite string-generated and that this is the worst possible case of state localization is of a more recent vintage [9] since it needed the support of the modular theory.

There is a very subtle aspect of modular localization which one encounters in the second Wigner representation class of massless finite helicity representations (the photon, graviton..class). Whereas in the massive case all spinorial fields $\Psi^{(A,\dot{B})}$ the relation of the physical spin s with the two spinorial indices follows the naive angular momentum composition rules

$$\begin{aligned} \left|A-\dot{B}\right| \leq s \leq \left|A+\dot{B}\right|,\ m>0 \\ s=\left|A-\dot{B}\right|,\ m=0 \end{aligned} \tag{19}$$

where the second line contains the hugely reduced number of spinorial descriptions for zero mass and finite helicity although in both cases the number of pointlike generators which are linear in the Wigner creation and annihilation operators [10].

By using the recourse of string-localized generators $\Psi^{(A,\dot{B})}(x,e)$ even in those cases where the representation has pointlike generators, one can even in the massless case return to the full spinorial spectrum as in the first line (19). These generators are covariant and "string-local"

$$\begin{aligned} U(\Lambda)\Psi^{(A,\dot{B})}(x,e)U(\Lambda) = D^{(A,\dot{B})}(\Lambda^{-1})\Psi^{(A,\dot{B})}(\Lambda x,\Lambda e) \\ \left[\Psi^{(A,\dot{B})}(x,e),\Psi^{(A',\dot{B}')}(x',e'\right]_{\pm} = 0,\ x+\mathbb{R}_{+}e >< x'+\mathbb{R}_{+}e' \end{aligned} \tag{20}$$

Here the unit vector e is the spacelike direction of the semiinfinite string and the last line expresses the spacelike fermionic/bosonic spacelike commutation. The best known illustration is the $(m=0,s=1)$ representation; in this case it is well-known that although a generating pointlike field strength exists, there is no pointlike vectorpotential. The modular localization approach offers as a substitute a stringlike covariant vector potential $A_{\mu}(x,e)$. In the case $(m=0,s=2)$ the "field strength" is a fourth degree tensor which has the symmetry properties of the Riemann tensor; in fact it is often referred to as the linearized Riemann tensor. In this case the string-localized potential is of the form $g_{\mu\nu}(x,e)$ i.e. resembles the metric tensor of general relativity. The consequences of this localization for a reformulation of gauge theory will be mentioned in a separate subsection.

A different kind of spacelike string-localization arises in d=1+2 Wigner representations with anomalous spin [44]. The amazing power of this modular

localization approach is that it preempts the spin-statistics connection in the one-particle setting, namely if s is the spin of the particle (which in d=1+2 may take on any real value) then one finds for the connection of the symplectic complement with the causal complement the generalized duality relation

$$\mathfrak{K}(\mathcal{O}') = Z\mathfrak{K}(\mathcal{O})'$$

where the square of the twist operator $Z = e^{\pi is}$ is easily seen (by the connection of Wigner representation theory with the two-point function) to lead to the statistics phase: Z^2 = statistics phase [44]. The fact that one never has to go beyond string localization (and fact, apart from those mentioned cases, never beyond point localization) in order to obtain generating fields for a QFT is remarkable in view of the many attempts to introduce extended objects into QFT.

It should be clear that modular localization which goes with real subspaces (or dense complex subspaces) unlike B-N-W localization cannot be connected with probabilities and projectors. It is rather related to causal localization aspects and the standardness of the K-space for a compact region is nothing else then the one-particle version of the Reeh-Schlieder property. It was certainly the kind of localization which Wigner was looking for because it represents the caminho real from representation theory into QFT. As will be seen in the next section it is also an important tool in the non-perturbative construction of interacting models.

2.5 Algebraic aspects of modular theory

A net of real subspaces $\mathfrak{K}(\mathcal{O}) \subset H_1$ for an finite spin (helicity) Wigner representation can be "second quantized"[27] via the CCR (Weyl) respectively CAR quantization functor; in this way one obtains a covariant $\mathcal{O}$-indexed net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ acting on the bosonic or fermionic Fock space $H = Fock(H_1)$ built over the one-particle Wigner space H_1. For integer spin/helicity values the modular localization in Wigner space implies the identification of the symplectic complement with the geometric complement in the sense of relativistic causality, i.e. $\mathfrak{K}(\mathcal{O})' = \mathfrak{K}(\mathcal{O}')$ (spatial Haag duality in H_1). The Weyl functor takes the spatial version of Haag duality into its algebraic counterpart. One proceeds as follows: for each Wigner wave function $\varphi \in H_1$ the associated (unitary) Weyl operator is defined as

$$\begin{aligned} Weyl(\varphi) &:= expi\{a^*(\varphi) + a(\varphi)\}, Weyl(\varphi) \in B(H) \qquad (21) \\ \mathcal{A}(\mathcal{O}) &:= alg\{Weyl(\varphi)|\varphi \in \mathfrak{K}(\mathcal{O})\}'', \ \ \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \end{aligned}$$

where $a^*(\varphi)$ and $a(\varphi)$ are the usual Fock space creation and annihilation operators of a Wigner particle in the wave function φ. We then define the von

[27] The terminology 2^{nd} quantization is a misdemeanor since one is dealing with a rigorously defined functor within QT which has little in common with the artful use of that parallellism to classical theory called "quantization". In Edward Nelson's words: (first) quantization is a mystery, but second quantization is a functor.

Neumann algebra corresponding to the localization region $\mathcal{O}$ in terms of the operator algebra generated by the functorial image of the modular constructed localized subspace $\mathfrak{K}(\mathcal{O})$ as in the second line. By the von Neumann double commutant theorem, our generated operator algebra is weakly closed by definition.

The functorial relation between real subspaces and von Neumann algebras via the Weyl functor preserves the causal localization structure and hence the spatial duality passes to its algebraic counterpart. The functor also commutes with the improvement of localization through intersections $\cap$ according to $K(\mathcal{O}) = \cap_{W\supset O}K(W),\ \mathcal{A}(\mathcal{O}) = \cap_{W\supset O}\mathcal{A}(W)$ as expressed in the commuting diagram

$$\begin{array}{ccc} \{K(W)\}_W & \longrightarrow & \{\mathcal{A}(W)\}_W \\ \downarrow\cap & & \downarrow\cap \\ K(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array} \tag{22}$$

Here the vertical arrows denote the tightening of localization by intersection whereas the horizontal ones denote the action of the Weyl functor.

The case of half-integer spin representations is analogous [43], apart from the fact that there is a mismatch between the causal and symplectic complements which must be taken care of by a *twist operator* $\mathcal{Z}$ and as a result one has to use the CAR functor instead of the Weyl functor.

In case of the large family of irreducible zero mass infinite spin representations in which the lightlike little group is faithfully represented, the finitely localized K-spaces are trivial $\mathfrak{K}(\mathcal{O}) = \{0\}$ and the *most tightly localized nontrivial spaces are of the form* $\mathfrak{K}(\mathcal{C})$ for $\mathcal{C}$ a *spacelike cone*. As a double cone contracts to its core which is a point, the core of a double cone is a *covariant spacelike semiinfinite string*. The above functorial construction works the same way for the Wigner infinite spin representation except that there are no nontrivial algebras which have a smaller localization than $\mathcal{A}(\mathcal{C})$ and there are no fields which are sharper localized than a semiinfinite string. Point- (or string-) like covariant fields are singular generators of these algebras i.e. they are operator-valued distributions. Stringlike generators, which are also available in the pointlike case, turn out to have an improved short distance behavior; whereas e.g. the short distance dimension of a free pointlike vectorfield is $sddA_\mu(x) = 2$ for its stringlike counterpart one has $sddA_\mu(x,e) = 1$ [10]. They can be constructed from the unique Wigner representation by so called intertwiners between the canonical and the many possible covariant (dotted-undotted spinor finite representations of the L-group) representations. The Euler-Lagrange aspect plays no role in these construction since the causal aspect of hyperbolic differential propagation are fully taken care of by modular localization.

A basis of local covariant field coordinatizations is then defined by Wick composites of the free fields. The string-like fields do not follow the classical behavior since already before introducing composites one has a continuous family of not classically intertwiners between the unique Wigner infinite spin representation and the continuously many covariant string interwiners. Their non-classical aspects, in particular the absence of a Lagrangian, are the reason

why their spacetime description in terms of semiinfinite string fields has been discovered only recently and not at the time of Jordan's field quantization.

Using the standard notation Γ for the second quantization functor which maps real localized (one-particle) subspaces into localized von Neumann algebras and extending this functor in a natural way to include the images of the $\mathfrak{K}(\mathcal{O})$-associated s,δ,j which are denoted by S,Δ,J one arrives at the Tomita Takesaki theory of the interaction-free local algebra $(\mathcal{A}(\mathcal{O}),\Omega)$ in standard position[28]

$$\begin{aligned} &H_{Fock}=\Gamma(H_1)=e^{H_1},\;\; \left(e^h,e^k\right)=e^{(h,k)} \\ &\Delta=\Gamma(\delta),\;\; J=\Gamma(j),\;\; S=\Gamma(s) \\ &SA\Omega=A^*\Omega,\;\; A\in\mathcal{A}(O),\;\; S=J\Delta^{\frac{1}{2}} \end{aligned} \tag{23}$$

With this we arrive at the core statement of the Tomita-Takesaki theorem which is a statement about the two modular objects Δ^{it} and J on the algebra

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O}))\equiv\Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it}=\mathcal{A}(\mathcal{O}) \\ J\mathcal{A}(\mathcal{O})J=\mathcal{A}(\mathcal{O})'=\mathcal{A}(\mathcal{O}') \end{aligned} \tag{24}$$

in words: the reflection J maps an algebra (in standard position) into its von Neumann commutant and the unitary group Δ^{it} defines an one-parametric automorphism-group σ_t of the algebra. In this form (but without the last geometric statement involving the geometrical causal complement $\mathcal{O}'$) the theorem hold in complete mathematical generality for standard pairs $(\mathcal{A},\Omega)$. The free fields and their Wick composites are "coordinatizing" singular generators of this $\mathcal{O}$-indexed net of algebras in the sense that the smeared fields $A(f)$ with $suppf\subset\mathcal{O}$ are (unbounded operators) affiliated with $\mathcal{A}(\mathcal{O})$.

In the above second quantization context the origin of the T-T theorem and its proof is clear: the symplectic disjoint passes via the functorial operation to the operator algebra commutant and the spatial one-particle automorphism goes into its algebraic counterpart. The definition of the Tomita involution S through its action on the dense set of states (guarantied by the standardness of $\mathcal{A}$) as $SA\Omega=A^*\Omega$ and the action of the two modular objects Δ,J (23) is part of the general setting of the modular Tomita-Takesaki theory; standardness is the mathematical terminology for the Reeh-Schlieder property i.e. the existence[29] of a vector $\Omega\in H$ with respect to which the algebra acts cyclic and has no "annihilators" of Ω. Naturally the proof of the abstract T-T theorem in the general setting of operator algebras is more involved.

The important property which renders this useful beyond free fields as a new constructive tool in the presence of interactions, is that for $(\mathcal{A}(W),\Omega)$ the antiunitary involution J depends on the interaction, whereas Δ^{it} continues to

[28]The functor Γ preserves the standardness i.e. maps the spatial one-particle standardness into its algebraic counterpart.

[29]In QFT any finite energy vector (which of course includes the vacuum) has this property as well as any nondegenerated KMS state. In the mathematical setting it is shown that standard vectors are "$\delta-$dense" in H.

be uniquely fixed by the representation of the Poincaré group i.e. by the particle content. In fact it has been known for some [11] time that J is related with its free counterpart J_0 through the scattering matrx

$$J = J_0 S_{scat} \tag{25}$$

This modular role of the scattering matrix as a relative modular invariant between an interacting theory and its free counterpart comes as a surprise. It is precisely this role which opens the way for an inverse scattering construction .

The physically relevant facts emerging from modular theory can be compressed into the following statements[30]

- *The domain of the unbounded operators $S(\mathcal{O})$ is fixed in terms of intersections of the wedge domains associated to $S(W)$; in other words it is determined by the particle content alone and therefore of a kinematical nature. These dense domains change with $\mathcal{O}$ i.e. the dense set of localized states has a bundle structure.*
- *The complex domains $DomS(\mathcal{O}) = K(\mathcal{O}) + iK(\mathcal{O})$ decompose into real subspaces $K(\mathcal{O}) = \overline{\mathcal{A}(\mathcal{O})^{sa}\Omega}$. This decomposition contains dynamical information which in case $\mathcal{O} = W$ reduces to the S-matrix (25). Assuming the validity of the crossing properties for formfactors, the S-matix fixes $\mathcal{A}(W)$ uniquely [13].*

The remainder of this subsection contains some comments about a remarkable constructive success of these modular methods. For this we need some additional terminology. Let us enlarge the algebraic setting by admitting unbounded operators with Wightman domains which are affiliated to $\mathcal{A}(\mathcal{O})$ and just talk about "$\mathcal{O}$-localized operators" when we do not want to distinguish between bounded and affiliated unbounded operators. We call an $\mathcal{O}$-localized operators a vacuum **p**olarization **f**ree **g**enerator (PFG) if applied to the vacuum it generated a one-particle state without admixture of a vacuum-polarization cloud. The following two theorems have turned out to be useful in a constructive approach based on modular theory.

Theorem: *The existence of an $\mathcal{O}$-localized PFG for a causally complete subwedge region $\mathcal{O} \subset W$ implies the absence of interactions i.e. the generating fields are (* a slight generalization of the Jost-Schroer theorem [45] which used the existence of pointlike covariant fields).

Theorem ([12]): *Modular theory for wedge algebras insures the existence of PFGs even in the presence of interactions. Hence the wedge region permits the best compromise between interacting fields and one-particle states.*

[30] Alain Connes would like to see a third spatial decomposition in that list namely the decomposition of K into a certain positive cone and its opposite. With such a requirement one could obtain the entire *algebra strucure from that of states.* This construction has been highly useful in Connes classification of von Neumann algebras, but it has not been possible to relate this with physical concepts.

Theorem ([12]): *Wedge localized PFGs with Wightman-like domain properties ("tempered" PFGs) lead to the absence of particle creation (pure elasic S_{scat}) which in turn is only possible in d=1+1 and leads to the factorizing models (which hitherto were studied in the setting of the bootstrap-formfactor program). The compact localized subalgebra $\mathcal{A}(\mathcal{O})$ have no PFGs and possess the full interaction-induced vacuum polarization clouds.*

Some additional comments will be helpful. The first theorem gives an intrinsic (i.e. not dependent on any Lagrangian or other extraneous properties) local definition of the presence of interaction, although it is not capable to differentiate between different kind of interactions (which would be reflected in the shapes of interaction-induced polarization clouds). The other two theorems suggest that the knowledge of the wedge algebra $\mathcal{A}(W) \subset B(H)$ may serve as a useful starting point for classifying and constructing models of LQP in a completely intrinsic fashion[31]. Knowing generating operators of $\mathcal{A}(W)$ including their transformation properties under the Poincaré group is certainly sufficient and constitutes the most practical way for getting the construction started.

All wedge algebras possess affiliated PFGs but only in case they come with reasonable domain properties ("tempered") they can presently be used in computations. This requirement only leaves models in d=1+1 which in addition must be factorizing (integrable); in fact the modular theory used in establishing these connections shows that there is a deep connection between integrability in QFT and vacuum polarization properties [12].

Tempered PFGs which generate wedge algebra for factorizing models have a rather simple algebraic structure. They are of the form

$$Z(x) = \int \left(\tilde{Z}(\theta) e^{-ipx} + h.c. \right) \frac{dp}{2p_0} \tag{26}$$

where in the simplest case $\tilde{Z}(\theta), \tilde{Z}^*(\theta)$ are one-component objects[32] which obey the Zamolodchikov-Faddeev commutation relations. In this way the formal Z-F device which encoded the two-particle S-matrix into the commutation structure of the Z-F algebra receives a profound spacetime interpretation. Like free fields they are on mass shell, but their creation and annihilation part obeys the Z-F commutation relations instead of the standard free field relations; as a result they are incompatible with pointlike localization but turn out to comply with wedge localization [13].

The simplicity of the wedge generators in factorizing models is in stark contrast to the richness of compactly localized operators e.g. of operators affiliated to a spacetime double cone $\mathcal{D}$ which arises as a relative commutant $\mathcal{A}(\mathcal{D}) = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$. The wedge algebra $\mathcal{A}(W)$ has simple generators and the full space of formal operators affiliated with $\mathcal{A}(W)$ has the form of an infinite series in the Z-F operators with coefficient functions $a(\theta_1, ...\theta_n)$ with analyticity

[31] In particular the above commuting square remains valid in the presence of interactions if one changes $\mathcal{O} \rightarrow W$.

[32] This case leads to the Sinh-Gordon theory and related models.

properties in a θ-strip

$$A(x) = \sum \frac{1}{n!} \int_{\partial S(0,\pi)} d\theta_1 ... \int_{\partial S(0,\pi)} d\theta_n e^{-ix\sum p(\theta_i)} a(\theta_1, ...\theta_n) \tilde{Z}(\theta_1) ... \tilde{Z}(\theta_1) \quad (27)$$

where for the purpose of a compact notation we view the creation part $\tilde{Z}^*(\theta)$ as $\tilde{Z}(\theta + i\pi)$ i.e. as coming from the upper part of the strip $S(0,\pi)$[33]. The requirement that the formal expressions of the form commutes with a series of the same kind but translated by a defines formally a subspace of operators affiliated with $\mathcal{A}(\mathcal{D}) = \mathcal{A}(W_a)' \cap \mathcal{A}(W)$. As a result of the simplicity of the $\tilde{Z}$ generators one can characterize these subspaces in terms of analytic properties of the coefficient functions $a(\theta_1, ...\theta_n)$ and one recognizes that they are identical [11] to the so-called formfactor postulates in the bootstrap-formfactor approach [46].

This is similar to the old Glaser-Lehmann-Zimmermann representation for the interacting Heisenberg field [47] in terms of incoming free field (in which case the spacetime dependent coefficient functions turn out to be on-shell restrictions of Fourier transforms of retarded functions), except that instead of the on-shell incoming fields one takes the on-shell $\tilde{Z}$ operators and the coefficient functions are the (connected part of the) multiparticle formfactors. As was the case with the GLZ series, the convergence of the formfactor series has remaind an open problem. So unlike perturbative series resulting from renormalized perturbation theory which have been shown to diverge even in models with optimal short distance behavior (even Borel resummability does not help), the status of the GLZ and formfactor series remains unresolved.

The main property one has to establish if one's aim is to secure the existence of a QFT with local observables, is the standardness of the double cone intersection $\mathcal{A}(\mathcal{D}) = \cap_{W \supset \mathcal{D}} \mathcal{A}(W)$. Based on nuclearity properties of degrees of freedom in phasespace discovered by Buchholz and Wichmann [48], Lechner has found a method within the modular operator setting of factorizing models which achieves precisely this [49][50]. For the first time in the history of QFT one now has a construction method which goes beyond the Hamiltonian- and measure-theoretical approach of the 60s [51]. The old approach could only deal with superrenormalizable models i.e. models whose basic fields did not have a short distance dimension beyond that of a free field.

At this point it is instructive to recall that although QFT has been the most successful of all physical theories as far as observational predictions are concerned, in comparison to those theories which already have a secure place in the pantheon of theoretical physics, it remains quite shaky concerning its mathematical and conceptual foundations. Looking at the present sociological situation it seems that the last past success which led to the standard model has generated an amnesia about foundational problems. Post standard model theories as string theory profited from this situation.

[33] The notation is suggested by the the strip analyticity coming from wedge localization. Of course only certain matrix elements and expectation values, but not field operators or their Fourier transforms, can be analytic; therefore the notation is symbolic.

The factorizing models form an interesting battle ground where problems, which accompanied QFT almost since its birth, have a good chance to receive a new push. The very existence of these theories, whose fields have anomalous trans-canonical short distance dimensions with interaction-dependent strengths, shows that there is nothing intrinsically threatening about singular short distance behavior. Whereas in renormalized perturbation theory the power counting rule only permits logarithmic corrections to the canonical (free field) dimensions, the construction of factorizing models starting from wedge algebras and their Z generators permits arbitrary high powers. That many problems of QFT are not intrinsic but rather caused by a particular method of quantization had already been suspected by the protagonist of QFT Pascual Jordan who, as far back as 1929, pleaded for a formulation "without (classic) crutches" [52]. The above construction of factorizing models which does not use any of the quantization schemes and in which the model does not even come with a Lagrangian name may be considered at the first realization of Jordan's plea at which he arrived on purely philosophically grounds.

The significant conceptual distance between QM and LQP begs the question in what sense the statement that QM is a nonrelativistic limit of LQP must be understood. By this we do not mean a manipulation in a Lagrangian or functional integral representation, but an argument which starts from the correlation functions or operator algebras of an interacting LQP. Apparently such an argument does not yet exist. One attempt in this direction could consist in starting from the known formfactors of a factorizing model as e.g. the Sinh-Gordon model and see the simplifications (vanishing of the vacuum polarization contributions) for small rapidity θ. An understanding in this sense would be an essential improvement of our understanding of the QM-QFT interface.

Since modular theory continues to play an important role in the two remaining sections, some care is required in avoiding potential misunderstandings. It is very crucial to be aware of the fact that by restricting the global vacuum state to, a say double cone algebra $\mathcal{A}(\mathcal{D})$, there is no change in the values of the global vacuum expectation values

$$(\Omega_{vac}, A\Omega_{vac}) = (\Omega_{\mathrm{mod},\beta}, A\Omega_{\mathrm{mod},\beta})\,, \quad A \in \mathcal{A}(\mathcal{D}) \tag{28}$$

where for the standard normalization of the modular Hamiltonian[34] $\beta = 1$. This notation on the right hand side means that the vacuum expectation values, if restricted to $A \in \mathcal{A}(\mathcal{D})$, fulfill an additional property (which without the restriction to the local algebra would not hold), namely the KMS relation

$$(\Omega_{\mathrm{mod},\beta}, AB\Omega_{\mathrm{mod},\beta}) = \left(\Omega_{\mathrm{mod},\beta}, B\Delta_{\mathcal{A}(\mathcal{O})}A\Omega_{\mathrm{mod},\beta}\right) \tag{29}$$

At this point one may wonder how a global vacuum state can turn into a thermal state on a smaller algebra without any thermal exchange taking place. The answer is that the in terms of $(A(\mathcal{D}), \Omega_{vac})$ canonically defined modular Hamiltonian H_{mod} with $\Delta = e^{-H_{\mathrm{mod}}}$ is very different from the original translative Hamiltonian H_{tr} whose lowest energy eigenstate defines the vacuum whereas H_{mod}

[34] The modular Hamiltonian lead to fuzzy motions within $\mathcal{A}(\mathcal{O})$ except in case of $\mathcal{O} = W$ when the modular Hamiltonian is identical to the boost generator.

is not the translative heat bath Hamiltonian but rather a thermal Hamiltonian which describes a (in general "fuzzy") movement inside $\mathcal{D}$ with the restricted vacuum representing a homogeneous thermal mean eigenstate. Its vanishing eigenvalue is not that of a ground state but sits in the middle of a symmetric two-sided spectrum. What has changed through the process of restriction is not the state but rather the way of looking at it: H_{mod} describes the dynamics of an "observer" confined to $\mathcal{D}$.

The thermal aspect of modular theory does of course not mean that one is converting a ground state into a thermal state in the sense of creating heat. As in the case of QM where the subdivision into an inside and outside the region via Born localization is primarily a Gedanken-act in order to create information theoretical entanglement from states which have a finite energy above the vacuum, the thermal aspect of the modular localization serves in first place to find an efficient description of the vacuum in terms of a smaller causally closed world. There is nothing more precise and intrinsic than saying that the restricted vacuum is a KMS state of a certain Hamiltonian even if there is no physical realization. In both cases one views the original state but from a different viewpoint.

But what about the Unruh effect which is associated with the Rindler wedge $\mathcal{O} = W$? Isn't this more than just a change of viewpoint? Yes and no. In order to create such a horizon the observer must be uniformely accelerated which requires feeding energy into the system. In other words the innocent looking restriction requires an enormous expenditure thus revealing in one example what is behind the innocent sounding word "restriction". An accepted rule of thumb is that only when the modular Hamiltonian describes a movement which corresponds to a diffeomorphism of spacetime is there a chance to think in terms of an Unruh kind of Gedankenexperiment. The modular situation is more advantageous in black hole situation where the position of event horizons is fixed by the metric. For example there exists a pure state on the Kruskal extension of the Schwarzschild solution (the Hartle-Hawking state) which restricted to the outside of the black hole describes the timelike Killing movement; in this case there is no doubt that the restriction corresponds to the natural time development in the world outside the black hole.

In fact there is a *continuous family of modular "Hamiltonians"* which are the generators the modular unitaries for sequences of included regions. The modular Hamiltonian of the larger region will of course spread the smaller localized algebra into the larger region. All these modular Hamiltonians have the two-sided symmetric spectrum which is typical for Hamiltonians in a KMS representation [3].

Besides the thermal description of restricted states there is one other macroscopic manifestation of vacuum polarization which has caused unbelieving amazement namely the cyclicity of the vacuum (the Reeh-Schlieder property) with respect to algebras localized in arbitrarily small spacetime region. The idea that by doing something in a small earthly laboratory for a say small fraction of a second by which a state "behind the moon" maybe approximated with arbitrary precision is certainly such a statement.

Both consequences of vacuum polarization, the thermal aspect of state restriction and the "state behind the moon property" are manifestations of a holistic behavior which is unheard of in QM. Instead of the division into an object to be measured on and the environment without which the modern quantum mechanical measurement theory can hardly be formulated, one has a situation which makes such a dichotomy illusory. By restricting to the inside one already specifies the dynamics on the causal disjoint, it governed by the same Hamiltonian. In the state behind the moon argument the difficulty in a system-environmen dichotomyt is even more palpable.

This is indeed an extremely surprising feature which goes considerably beyond the kinematical change caused by entanglement as the result of the quantum mechanical division into measured system and environment. It is this dependence of the reduced vacuum state on the localization region inside which it is tested with localized algebras which raises doubts about what one really associates with the non-fleeting persistent properties of a material substance. The monad description in the next section strengthens this little holistic aspect of LQP.

In both cases QM as well as QFT the entanglement comes about by a different ways of looking at the system and not by changing intrinsic properties, but the *thermal entanglement* of QFT associated with modular localization is more spectacular than the (*"cold") information-theoretic kind of entanglement* of QM associated with Born localization.

As we have seen the thermal aspects of modular localization are very rich from an epistemic viewpoint. The ontic content of these observations is quite weak; it is only when the (imagined) causal localization horizons passes from a Gedanken objects to a (real) event horizons through the curvature of spacetime that the fleeting aspect of observers horizons becomes an ontic property of spacetime as in black holes. But even if one's main interest is to do black hole physics, it is wise to avoid a presently popular "shut up and compute" attitude and to understand the conceptual basis in LQP of the thermal aspect of localization and the peculiar thermal entanglement which contrasts the information-theoretical quantum mechanical entanglement. Not caring about these conceptual aspects one may easily be drawn into a fruitless and protractive arguments as it happened (and is still happening) with the entropy/information loss issue. These problems are connected to an insufficient conceptual understanding of QFT by identifying it with some sort of relativistic QM.

Up to now the terminology "localization" was used both for states and for subalgebras. Whereas in the absence of interactions it is true that they are synonymous in the sense that when a dense subspace of $\mathcal{O}$ localized states results from the application of an $\mathcal{O}$ localized algebra onto the vacuum, such a close relation between algebraic and spatial localization breaks down in the presence of interactions. It is perfectly conceivable to have a theory with "topological charges" [3] which by definition are not compactly localizable but rather spacelike cone localizable (in terms of generating fields semiinfinite string-localizable). In that case only the neutral observable algebra has the usual compact localizability. property whereas the charge-carrying part of the total algebra is at best

localizable in the sense of semiinfinite strings (the field generators) of such an algebra. But massive charged states can always be written in terms of pointlike state-valued distributions; the modular decomposition theory of representations of the Poincaré group prevents pointlike generation only in the presence of infinite spin representations.

2.6 String-localization and gauge theory

Zero mass fields of finite helicity play a crucial role in gauge theory. Whereas in classical gauge theory a pointlike massless vectorpotential is not unphysical because otherwise it would contradict classical principles but rather because it is a pure auxiliary construct in the setting of Maxwell's theory, The situation changes radically in QFT because a covariant zero mass vectorpotential can only exist in form of a string-localized field, a covariant pointlike localized contradicts the Hilbert space structure of QT. Nevertheless there exists an indefinite formalism with additional "ghost degrees of freedom", the Gupta Bleuler formalism in QED and the BRST ghost formalism in QCD, which permits to return to physical quantities in a Hilbert space setting by what is interpreted as the quantum version of gauge invariance.

This only has been shown in perturbation theory and it would not be over pessimistic to expect that manipulations which depend on convergence have no meaning outside the Hilbert space topology. But there is actually a much stronger physical reason having to do with localization why the gauge theory does not give the full insight into QED and this certainly worsens in passing to QCD. In QED the physically most important objects are the electrically charged fields. Since they are nonlocal physical object they are not part of the perturbative gauge setting which aims at the local gauge invariant; in some sense they are nonlocal gauge invariant fields but this is just another way of saying that their construction requires ingenuity and luck because the formalism of perturbation theory does not lead to a computational rule for charged fields.

Using a version of perturbation theory which was especially designed for charged fields, Steinmann [53] succeeded to make some headway on this problem. Related to this nonlocality aspect is the subtle relation of electrically charged fields to charged particles is their involved infrared aspect; a charged particle even after a long time of having left the scattering region will never be without an infinite cloud of infrared real (not virtual!) photons whose energy is below the registering resolution and which therefore remain "invisible". This makes charge particles "infraparticles" i.e. objects whose scattering theory does not lead to scattering amplitudes but only the inclusive cross sections.

The problem of nonlocal fields becomes much more series in theories involving vectorfields coupled among themselves. Whereas one believes to have a physical understanding of the local gauge invariant composites whose perturbation expansion has incurable infrared divergencies, there is not the slightest idea what is at the place of the charged fields. For four decades one uses nice words as quark and gluon confinement well aware that, different from QM, QFT has no mechanism which can enclose quantum matter in a vault, rather the only modal-

ity to arrive at "invisibility" is through still stronger delocalization, involving not only the would be charged matter but even the selfinteracting vectorpotentials. There is little chance that this can be done "by hand", as in the case of QED.

The only alternative to the present gauge method in which the pointlike localization of covariant vectorpotentials is paid for by the unphysical ghost formalism and the subsequent restriction to local observables is to work with string-localized potentials $A_\mu(x,e)$. This poses completely new and still largely unsolved problems. But before commenting on this new task, it is helpful to delineate what one expects of such an alternative approach.

Superficially such string-localized fields seem to be indistinguishable from the axial gauge; here as there the conditions $\partial^\mu A_\mu(x) = 0 = e^\mu A_\mu(x)$ are obeyed. In the axial gauge interpretation the e is a gauge parameter and does not participate in Lorentz transformations whereas in the formula for a string-localized field the spacelike unit vector transforms as a string direction. The distance between the two concepts increases when one passes from free fields to their perturbative interactions. It is well known that the axial gauge formalism fails on its infrared divergencies; there has been no successful computation involving loops. The string-localized approach on the other hand requires the computation of perturbative correlation functions with variable string direction

$$\left\langle 0 \left| A_{\mu_1}(x_1,e_1)...A_{\mu_n}(x_n,e_n)\psi(y_1)...\bar{\psi}(y_m) \right| 0 \right\rangle \tag{30}$$

Also at the end of the calculation when one extracts the physical result one must study the infrared behavior of coalescing strings, it is important to keep the directions e_i as variable string directions (and not as a fixed gauge parameter) and consider the correlations as distributions in x as well as in e. Only in this way one has a chance to handle the infrared divergencies of these theories and understand their physical role. Note that the implicit dependence of the matter field ψ and $\bar{\psi}$ on the $e's$ of the inner vectorpotential lines has been omitted for obvious reasons of compactness of notation.

The infrared singularity will appear at the end when all $e's$ coalesce but then it is not just an unspecified divergence but rather takes on the appearance of a short distance limit in a one lower dimensional de Sitter space (the space of spacelike directions) This spacetime interpretation for the infrared divergencies is missing in the axial gauge setting.

Although the power counting behavior of quadrilinear interactions of string-localized fields is not worse than that of pointlike interactions in the ghost formalism of gauge theory, there is a serious problem with the perturbative Epstein Glaser [54] iteration. In the pointlike case the knowledge of the nth order determines the n+1 order up to a term on the total diagonal which limits the freedom to the addition of pointlike composites. The presence of string-like fields invalidates this argument; even if all strings lie in one spacelike hypersurface, the freedom is larger than that allowed by the total diagonal. What one hopes for is that the freedom can be described in terms of some string composites, whatever that means.

For QED one is in the fortunate position of being able to compare whatever this new string-localized perturbation leads to with the before mentioned gauge theoretic calculation where the nonlocal aspect of electrically charged operators needs to be added by hand. In the QCD case it would be unreasonable to expect that nonlocal physical operators do not exist, but nobody in the 4 decades since the inception of gauge QFT came up with a reasonable suggestion as to how such object could look like. This problem has been studied on the lattice, but if lattice gauge theory was not even able to describe the charge-carrying infraparticle fields of QED what hope is left to understand the structure of QCD beyond its pointlike-localized observable content? It seems that the only possibility to make headway on or most important problems of particle physics is to go after a theory of interacting semiinfinite string-localized fields.

Although for massive particles there is no structural reason to introduce string-localized fields, viz. the free massive vectorpotential $A_\mu(x)$ which exists in the Wigner-Fock space, the short distance dimension of such fields increase with increasing spin so that already a s=1 field leads to sdd=2 and hence does not allow a fourth power coupling of fields within the power counting limit. A string-localized massive vectorpotential has sdd=1 and would therefore lead to quadrilinear interactions within this limit. Assuming that the Epstein-Glaser iteration has an extension to string-localized fields one could hope for a better intrinsic understanding of the Schwinger Higgs screening mechanism than within the present gauge setting. In particular one may finally understand how the presence of a scalar particle renders the whole theory pointlike local so that the use of a string-localized vector potential had the sole purpose of enabling a renormalizable interaction.

2.7 Building LQP via *positioning of monads* in a Hilbert space

We have seen that modular localization of states and algebras is an intrinsic i.e. field-coordinatization-independent way to formulate the kind of localization which is characteristic for QFT. It is deeply satisfying that it also leads to a new constructive view of QFT.

Definition (Wiesbrock [55])**:** *An inclusion of standard operator algebras* $(\mathcal{A}\subset\mathcal{B},\Omega)$ *is "modular" if* $(\mathcal{A},\Omega)$ *and* $(\mathcal{B},\Omega)$ *are standard and* $\Delta_{\mathcal{B}}^{it}$ *acts like a compression on* $\mathcal{A}$ *i.e.* $Ad\Delta_{\mathcal{B}}^{it}\mathcal{A}\subset\mathcal{A}$*. A modular inclusion is said to be standard if in addition the relative commutant* $(\mathcal{A}'\cap\mathcal{B},\Omega)$ *is standard. If this holds for* $t<0$ *one speaks about a -modular inclusion.*

The study of inclusions of operator algebras has been an area of considerable mathematical interest. Particle physics uses 3 different kind of inclusions; besides the modular inclusions which play the principal role in this section there are *split inclusions* and inclusions with conditional expectations or using the name of their creator Vaughn *Jones inclusions.* Split inclusions play an important role in structural investigation and are indispensable in the study of thermal aspects of localization notably localization entropy (see next chapter). Jones inclusions result from reformulating the DHR theory of superselection sectors

which in its original formulation uses the formalism of localized endomorphisms of observable algebras.

The important achievement of that theory is that the local system of observables has enough structure in order to complement the theory with its charged fields and their inner symmetries such that the original observables reemerge as the fixed point under this symmetry. This projection is accomplished in terms of a conditional expectation. The prototype of a conditional expectation in the conventional formulation of QFT (based on the use of charge-carrying fields) is the averaging over the compact internal symmetry group with its normalized Haar measure ($U(g)$ denotes the representation of the internal symmetry group)

$$\mathcal{A} = \int d\mu(g) AdU(g)\mathcal{F} \tag{31}$$
$$E : \mathcal{F} \xrightarrow{\mu} \mathcal{A},\ E^2 = E$$

i.e. the conditional expectation E projects the (charged) field algebra $\mathcal{F}$ onto the (neutral) observable algebra $\mathcal{A}$ and such inclusions which do not change the localization are therefore related to internal symmetries as opposed to spacetime symmetries.

Inclusions $\mathcal{A} \subset \mathcal{B}$ with conditional expectation $E(\mathcal{B})$ cannot be modular and the precise understanding why this is the case discloses interesting insights. According to a theorem of Takesaki [58] the existence of a conditional expectation is tantamount to the modular group of the smaller algebra being equal to the restriction of that of the bigger. Hence the natural generalization of this situation is that the group $Ad\Delta_{\mathcal{B}}^{it}$ of the larger algebra acts on $\mathcal{A}$ for either $t < 0$ or for $t > 0$ as a compression (endomorphism) and the absence of a conditional expectation. Intuitively speaking modular inclusions are too deep in order to allow conditional expectations. Continuing this line of speculative reasoning one would expect that as "flat" inclusions with conditional expectations are related to inner symmetries, "deep" inclusions of the modular kind lead to spacetime symmetries.

Surprisingly this rough guess turns out to be amazingly correct. The main aim of modular inclusions is really to *generate spacetime symmetry* as well as the *net of spacetime indexed algebras* which are covariant under these symmetries. This is done as follows: from the two modular groups $\Delta_{\mathcal{B}}^{it}, \Delta_{\mathcal{A}}^{it}$ one can form a unitary group $U(a)$ which together with the modular unitary group of the smaller algebra $\Delta_{\mathcal{B}}^{it}$ leads to the commutation relation $\Delta_{\mathcal{B}}^{it}U(a) = U(e^{-2\pi t}a)\Delta_{\mathcal{B}}^{it}$ which characterizes the 2-parametric translation-dilation (Anosov) group. One also obtains a system of local algebras by applying these symmetries to the relative commutant $\mathcal{A}' \cap \mathcal{B}$. From these relative commutants one may form a new algebra $\mathcal{C}$

$$\mathcal{C} \equiv \overline{\bigcup_t Ad\Delta_{\mathcal{B}}^{it}(\mathcal{A}' \cap \mathcal{B})} \tag{32}$$

In general $\mathcal{C} \subset \mathcal{B}$ and we are in a situation of a nontrivial inclusion to which the Takesaki theorem is applicable (the modular group of $\mathcal{C}$ is the restriction of that of $\mathcal{B}$) which leads to a conditional expectation $E : \mathcal{B} \to \mathcal{C}$; $\mathcal{C}$ may also be trivial.

The most interesting situation arises if the modular inclusion is *standard* i.e. all three algebras $\mathcal{A}, \mathcal{B}, \mathcal{A}' \cap \mathcal{B}$ are standard with respect to Ω; in that case we arrive at a chiral QFT.

Theorem: (Guido,Longo and Wiesbrock [59]) *Standard modular inclusions are in one-to-one correspondence with strongly additive chiral LQP.*

Here chiral LQP is a net of local algebras indexed by the intervals on a line with a Moebius-invariant vacuum vector and *strongly additive* refers to the fact that the removal of a point from an interval does not "damage" the algebra i.e. the von Neumann algebra generated by the two pieces is still the original algebra. One can show via a dualization process that there is a unique association of a chiral net on $S^1 = \dot{\mathbb{R}}$ to a strongly additive net on $\mathbb{R}$. Although in our definition of modular inclusion we have not said anything about the nature of the von Neumann algebras, it turns out that the very requirement of the inclusion being modular forces both algebras to be hyperfinite type III_1 factor algebras. The closeness to Leibniz's idea about (physical) reality of originating from relations between monads (with each monad in isolation of being void of individual attributes) more than justifies our choice of name; besides that "monad" is much shorter than the somewhat long winded mathematical terminology "hyperfinite type III_1 Murray-von Neumann factor algebra". The nice aspect of chiral models is that one can pass between the operator algebra formulation and the construction with pointlike fields without having to make additional technical assumptions[35]. Another interesting constructive aspect is that the operator-algebraic setting permits to establish the existence of algebraic nets in the sense of LQP for all $c < 1$ representations of the energy-momentum tensor algebra. This is much more than the vertex algebra approach is able to do since that formal power series approach is blind against the dense domains which change with the localization regions.

The idea of placing the monad into modular positions within a common Hilbert space may be generalized to more than two copies. For this purpose it is convenient to define the concept of a *modular intersection* in terms of modular inclusion.

Definition (Wiesbrock [55]): *Consider two monads A and B positioned in such a way that their intersection $A \cap B$ together with A and B are in standard position with respect to the vector $\Omega \in H$. Assume furthermore*

$$
\begin{aligned}
&(\mathcal{A} \cap \mathcal{B} \subset \mathcal{A}) \text{ and } (\mathcal{A} \cap \mathcal{B} \subset \mathcal{B}) \text{ are } \pm mi \\
&J_{\mathcal{A}} \lim_{t \to \mp} \Delta_{\mathcal{A}}^{it} \Delta_{\mathcal{B}}^{-it} J_{\mathcal{A}} = \lim_{t \to \mp} \Delta_{\mathcal{B}}^{it} \Delta_{\mathcal{A}}^{-it}
\end{aligned} \tag{33}
$$

then (A, B, Ω) is said to have the $\pm$ modular intersection property ($\pm$ mi).

It can be shown that this property is stable under taking commutants i.e. if $(\mathcal{A}, \mathcal{B}, \Omega) \pm mi$ then $(\mathcal{A}', \mathcal{B}', \Omega)$ is $\mp mi$.

[35] The group theoretic arguments which go into that theorem [66] seem to be available for any conformal QFT.

The minimal number of monads needed to characterize a 2+1 dimensional QFT through their modular positioning in a joint Hilbert space is three. The relevant theorem is as follows

Theorem: (Wiesbrock [56]) *Let A_{12},A_{13} and A_{23} be three monads*[36] *which have the standardness property with respect to $\Omega \in H$. Assume furthermore that*

$$\begin{aligned} &(\mathcal{A}_{12}, \mathcal{A}_{13}, \Omega) \ is\ -mi \\ &(\mathcal{A}_{23}, \mathcal{A}_{13}, \Omega) \ is\ +mi \\ &(\mathcal{A}_{23}, \mathcal{A}_{12}', \Omega) \ is\ -mi \end{aligned} \tag{34}$$

then the modular groups Δ_{12}^{it}, Δ_{13}^{it} and Δ_{23}^{it} generate the Lorentz group $SO(2,1)$.

Extending this setting by placing an additional monad $\mathcal{B}$ into a suitable position with respect to the $\mathcal{A}_{ik}$ of the theorem, one arrives at the Poincaré group $\mathcal{P}(2,1)$ [57]. The action of this Poincaré group on the four monads generates a spacetime indexed net i.e. a LQP model and all LQP have a monad presentation.

To arrive at d=3+1 LQP one needs 6 monads. The number of monads increases with the spacetime dimensions. Whereas in low spacetime dimensions the algebraic positioning is natural within the logic of modular inclusions, in higher dimensions it is presently necessary to take some additional guidance from geometry, since the number of possible modular arrangements for more than 3 monads increases.

We have presented these mathematical results and used a terminology in such a way that the relation to Leibniz philosophical view is highly visible.

Since this is not the place to give a comprehensive account but only to direct the attention of the reader to this (in my view) startling conceptual development in the heart of QFT.

Besides the radically different conceptual-philosophical outlook on what constitutes QFT, the modular setting offers new methods of construction. It turns out that for that purpose it is more convenient to start from one monad $\mathcal{A} \subset B(H)$ and assume that one knows the action of the Poincaré group via unitaries $U(a,\Lambda)$ on $\mathcal{A}$. If one interprets the monad $\mathcal{A}$ as a wedge algebra $\mathcal{A} =$ than the Poincaré action generates a net of wedge algebras $\{\mathcal{A}(W)\}_{W\in\mathcal{W}}$. A QFT is supposed to have local observables and if the double cone intersections[37] $\mathcal{A}(D)$ turn out to be trivial (multiples of the identity algebra) the net of wedge algebras does not leads to a QFT. This is comparable to the non-existence of a QFT which was to be associated via quantization to a Lagrangian. If however these intersections are nontrivial than the ontological status is much better than that we would have an existence proof which is much more than a non-converging renormalized perturbative series of which we do not know if and how it is related to a QFT. There are of course two obvious sticking points: (1) to find

[36] As in the case of a modular inclusion, the monad property is a consequence of the modular setting. But for the presentation it is more convenient and elegant to talk about monads from the start.

[37] Double cones are the typical causally complete compact regions which can be obtained by intersecting wedges.

Poincaré-covariant generators of $\mathcal{A}(W_0)$ and (2) a method which establishes the non-triviality of intersections of wedge algebras and leads to formulas for their generating elements.

As was explained in the previous section, both problems have been solved within a class of factorizing models. Nothing is known about how to address these two points in the more general setting i.e. when the tempered PFG are not available. Perhaps one should first test a perturbative version of this program which is expected to incorporate more possibilities than the perturbation theory based on pointlike fields since wedge-localized generators are free of those ultraviolet aspects which come from pointlike localization. The dynamic input in that case would not be a Lagrangian but rather the lowest order (tree-approximation) S-matrix interpreted as the in-out formfactor of the identity operator.

2.8 The split inclusion

There is one property of LQP which is indispensable for understanding how the quantum mechanical tensor factorization can be reconciled with modular localization: the *split property.*

Definition: *Two monads A, B are in a split position if the inclusion of monads $\mathcal{A} \subset \mathcal{B}'$ admits an intermediate type I factor $\mathcal{N}$ such that $A \subset N \subset B'$*

Split inclusions are very different from modular inclusions or inclusions with conditional expectations (Jones-DHR). The main property of a split inclusion is the existence an $\mathcal{N}$-dependent unitarily implemented isomorphism of the $\mathcal{A}, \mathcal{B}$ generated operator algebra into the tensor product algebra

$$\mathcal{A} \vee \mathcal{B} \to \mathcal{A} \otimes \mathcal{B} \subset \mathcal{N} \otimes \mathcal{N}' = B(H) \tag{35}$$

The prerequisite for this factorization in the LQP context is that the monads commute, but it is well-known that local commutativity is not sufficient, the counterexample being two double cones which touch each other at a spacelike boundary [3]. As soon as one localization region is separated from the other by a (arbitrary small) spacelike security distance, the interaction-free net satisfies the split property under very general conditions. In [60] the relevant physical property was identified in form of a phase space property. Unlike QM, the number of degrees of freedom in a finite phase space volume in QFT is not finite, but its infinity is quite mild; it is a nuclear set for free theories and this nuclearity requirement[38] is then postulated for interacting theories. The physical reason behind this nuclearity requirement is that it allows to show the existence of temperature states once one knows that a QFT exists in the vacuum representation.

The split property for two securely causally separated algebras has a nice physical interpretation. Let $\mathcal{A} = \mathcal{A}(\mathcal{O}),\ \mathcal{B}' = \mathcal{A}(\check{\mathcal{O}}),\ \mathcal{O} \subset \check{\mathcal{O}}$. Since $\mathcal{N}$ contains $\mathcal{A}$ and is contained in $\mathcal{B}'$ (but without carrying the assignment of a localization between $\mathcal{O}$ and $\check{\mathcal{O}}$), one may imagine $\mathcal{N}$ as an algebra which shares the sharp localization with $\mathcal{A}(\mathcal{O})$ in $\mathcal{O}$, but its localization in the "collar" between $\mathcal{O}$ and $\check{\mathcal{O}}$

[38] A set of vectors is nuclear if it is contained in the range of a trace class operator.

is "fuzzy" i.e. the collar subalgebra is like a "fog" which does not really occupy the collar region. This is precisely the region which is conceded to the vacuum polarization cloud in order to spread and thus avoid the infinite compression into the surface of a sharply localized monad. If we take a sequence of $\mathcal{N}$'s which approach the monad $\mathcal{A}$ the vacuum polarization clouds become infinitely large so that no direct definition of e.g. their energy or entropy is possible.

The inclusion of the tensor algebra of monads into a type I tensor product (35) looks at first sight like a déjà vu of QM tensor factorization, but there are interesting and important differences. In QM the tensor factorization obtained from the Born localization projector and its complement is automatic since the vacuum of QM (or the ground state of a quantum mechanical zero temperature finite density system) tensor factorizes. In QFT the vacuum does not tensor factorize at all but there are other "split vacuum" states in the Hilbert space which emulate a vacuum in the sense that expectation values of operators in $\mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\check{\mathcal{O}}')$ factorize in the split vacuum

$$\langle 0_{split} \left| AB \right| 0_{split} \rangle = \langle 0 \left| A \right| 0 \rangle \langle 0 \left| B \right| 0 \rangle , \ A \in \mathcal{A}(\mathcal{O}), B \in \mathcal{A}(\check{\mathcal{O}}') \tag{36}$$

But there is a huge conceptual difference to the quantum mechanical Born factorization of the "nothing" state. The splitting process requires the supply of energy since the split vacuum has infinite vacuum polarization (with finite mean energy) in the collar region which is spacelike to $\mathcal{O} \vee \check{\mathcal{O}}'$. If one agrees that the physical states of QFT are the states with a finite particle number than a split vacuum which has finite mean energy but infinite particle number does not look very realistic.

The problem of physical realizability is not given much attention in foundational discussions of QM. But in QFT this issue is more serious since the situations are much more counter-intuitive.as was shown before with the state behind the moon argument for the global vacuum. This property is lost in a split vacuum state but it is unclear how such states can be prepared and monitored.
.

Most foundational properties of QM as violation of Bell's inequalities, the Schroedinger cat property and many other strong deviations from classical reality can be experimentally verified. This is not possible for the vacuum polarization caused properties which result from modular localization since macroscopic manifestations are too small.. A typical example is the Unruh effect i.e. the thermal manifestation of a uniformly accelerated particle counter in the global vacuum where the temperature created by an acceleration of 1m/sec is $10^{-19}K$ too small for ever being registered. But for the perception of the reality which underlies LQP the difficulty in registering such effects does not diminish their importance.

The characterization of the restriction of the global vacuum to a local algebra in terms of a thermal state for a modular Hamiltonian holds independent of whether the local algebra is a sharply localized monad $\mathcal{A}(\mathcal{O})$ or a type I factor $\mathcal{N}$ as above in the splitting construction. The only difference is that the in the second case the KMS state is also a Gibbs state i.e. the Hamiltonian has

a discrete spectrum (in case $\check{\mathcal{O}}$ is compact). This thermal reinterpretation of reduced states does not only hold for the vacuum but applies to all states which are of physical relevance in particle physics i.e. to all finite energy states for which the Reeh-Schlieder theorem applies.

Since KMS states on type I factors are Gibbs states, there exists a density matrix. Therefore these Gibbs state can have a finite energy and entropy content which for monads is impossible. But a monad may be approximated by a sequence of type I factors in complete analogy to the thermodynamic limit. In fact the thermodynamic limit is the only place where a monad algebra appears in a QM setting; an indication that this limit is accompanied by a qualitative change is the fact that one looses the density matrix nature of the Gibbs state which changes to a more singular KMS state which simply does not exist on quantum mechanical type I algebras. A related fact is the breakdown of the tensor factorization into physical degrees of freedom and their "shadow world" which is the basis of the "Thermofield formalism", monad algebras simply do not allow such a tensor factorization.

The structural difference can be traced back to the modular Hamiltonians, whereas for monads the modular Hamiltonian has continuous spectrum (a typical example is a quantum mechanical Gibbs state box Hamiltonian in the $V \rightarrow \infty$ thermodynamic limit representation) and hence an ill-defined (infinite) value of energy and entropy, this is not the case for the $\mathcal{N}$-associated density matrix constructed from the split situation. So the way out is obvious: just imitate the thermodynamic limit by constructing a sequence of type I factors (a "funnel") $N_i \supset \mathcal{A}(\mathcal{O})$ (by tightening the split) which converge from the outside towards the monad (equivalently one may approximate from the inside). This is precisely what will be done for the computation of the localization entropy in the next section.

making the split limit in which a monad (the limiting KMS equilibrium situation in the standard heat bath setting) is approximated by a sequence of finite volume Gibbs states for which energy and entropy are finite and only diverge in the "monad limit". Indeed this will be the main idea or the derivation of the entropical area law in the next section.

In the above form the monad-positioning aims at characterizing LQP in Minkowski spacetime. This begs the question whether there is a generalization to curved spacetime. A very special exploratory attempt in this direction would be to investigate whether the $\mathrm{Diff}(S^1)$ symmetries beyond the Moebius group in chiral theories have a modular origin in terms of positioning monads relative to reference states. Since the extended chiral theories which originate from null-surface holography (and not from chiral projections of a two-dimensional conformal QFT) seem to have great constructive potential, this question may also be of practical interest. There are indications that this can be done if one relaxes on the idea of a universal vacuum reference state and allows "partial vacua" i.e. modular defined states which have geometric properties only on certain subalgebras (work in progress).

I expect that by pursuing the algebraization of QFT in CST via the positioning of monads to its limits one will learn important lessons about the true

QFT/QG interface. A conservative approach which explores unknown aspects of QFT while staying firmly rooted in known principles seems to be the most promising path in the present situation.

3 Problematization of the QFT-QG interface

In the previous section we outlined a radical new way of interpreting the conceptual content of QFT by highlighting those structure which are most different from QM. However in doing this we paid attention that this new way is at the same time conservative vis-à-vis the underlying physical principles. In certain cases, as 2-dim. integrable models, where one finds sufficiently well behaved generators of wedge algebras with simple vacuum polarization properties, one arrives at a nonperturbative scheme for the construction of models. The interesting aspect of these constructions, besides the fact that they are the first existence proofs for strictly renormalizable models[39], is that the umbilical quantization cord with classical physics has been cut, i.e. for the first time the more fundamental QFT was constructed without any reference to a quantization parallelism (Lagrangians, Functional Integrals,..).

We also indicated how the positioning of monads could be useful for a better future understanding of the interface between QFT in CST and QG. In this section more light will be shed on the thermal manifestations of causal localization. In particular two recent results about presently hotly debated topics will be presented namely the *universal area law of localization entropy,* which shifts[40] the interface between LQP and the elusive QG, and an intrinsic definition of the *energy density in cosmological reference states* (vacuum-like states in cosmological models) in the setting of QFT in CST.

3.1 Some history of area behavior of localization induced vacuum polarization

The phenomenon of vacuum polarization has been the point of departure of many metaphors of which the steaming broil is perhaps the best known because it occasionally even entered textbooks. In order to support this image its was claimed that a short time violation of the energy conservation is supported by the uncertainty relation. A less metaphoric view comes from locally "banging" on the vacuum i.e. applying a compactly localized observable to it. Such a banged state is characterized by its n-particle matrix elements for all n and these n-particle vacuum polarization components are in turn special boundary values of an analytic n-particle master function whose different out-in particle distributions obtained from the vacuum polarization component by crossing are

[39]The models constructed in the 60s were superrenormalizable i.e. of a short distance type which does not occur in d=4.

[40]This applies only to people who thought that one needs QG in order to understand the area law of black hole entropy.

the formfactors of A.

$$
\begin{aligned}
A|0\rangle &\simeq \{\langle p_1, ...p_n| A |0\rangle\}_n \\
&\overset{cros\sin g}{\rightarrow} \{\langle -p_1, ... - p_k| A |p_{k+1}, ...p_n\rangle\}_n
\end{aligned}
$$

where the negative mass shell momenta -p denotes the analytic continuation which is part of the crossing process. The crossing property follows from the fact that the extended wedge algebra

$$
\mathcal{A}_{ext}(W) \equiv \mathcal{A}_{out}(W) \vee \mathcal{A}(W) \vee \mathcal{A}_{in}(W)
$$

is a subalgebra which shares the boost Hamiltonian and that the restriction of the vacuum to $\mathcal{A}_{ext}(W)$ is a boost-KMS state. In order to generate a local bang on the vacuum one indeed creates a soup of particles and although the expectation of the energy in such a bang state is finite a bang with sharp localization has no limitation on high particle momenta in the formfactors of a localized operator. With other words none of the formfactors of such an operator vanishes in any region of momenta of multiparticle space with the only restriction coming from charge superselection rules.

Whereas in QM, relativistic or not, one has great liberty in manipulating interactions so that almost any outcome can be accommodated, this is not the case in QFT. This tightness even show up in theorems about the S-matrix as Aks theorem: in a 4-dimensional QFT nontrivial elastic scattering is not possible without the presence of inelastic components [12]. For the formfactors the previous statement in a more popular jargon permits a stronger and more general formulation in terms of a benevolent Murphy's law: all couplings of local operators to other channels (in the case of formfactors multiparticle channels) which are not forbidden by superselection rules actually do occur. Of course one needs to bang onto the vacuum, there is no "boiling soup" in a an inertial frame without heating the vacuum stove. The formfactor aspect of a local operator is perhaps the best QFT illustration of Murphy's law to particle physics.

Of course vacuum polarization as a concomitant phenomenon of QFT was discovered a long time before the role of locality it became clear. It is interesting to reformulate Heisenberg's observation in a lightly more modern context by defining partial charges by limiting the charged region with the help of smooth test function. In Heisenberg's more formal setting the partial charge of a free conserved current in a spatial volume V is defined as

$$
\begin{aligned}
Q_V &= \int_V j_0(x,t) d^3x \qquad (37) \\
j_\mu(x,t) &=: \phi^*(x,t) \overleftrightarrow{\partial}_\mu \phi(x,t) :
\end{aligned}
$$

Introducing a momentum space cutoff, the norm of $Q_V|0\rangle$ turns out to diverge quadratically which together with the dimensionlessness of Q is tied to the area proportionality. Hence already on the basis of a crude dimensional reasoning one finds an area proportionality of vacuum polarization. The cutoff was the

prize to pay for ignoring the singular nature of the current which is really not an operator but rather an operator-valued distribution.

The modern remedy is to take care of the divergence by treating the singular current as an operator-valued distribution. Such calculations have been done in the 60s by using spacetime test functions which regularize the delta function at coalescing times and are equal to one inside the ball with radius R and fall off to zero smoothly between R and $R+\Delta R$. Using the conservation law of the current one can then show that the action of the regularized partial charge on the vacuum is compressed to the shell $(R, R+\Delta R)$ and diverges quadratically with $\Delta R \rightarrow 0$ i.e. As expected, the vacuum fluctuations vanish weakly as $R \rightarrow \infty$ (even strongly by enlarging the time smearing support together with R) i.e. the limit converges independent of the special test function weakly[41] to the global charge operator

$$\lim_{R\rightarrow\infty} \int f_R(\vec{x})g(t)j_0(x,t)d^4x = Q \tag{38}$$

The problem of localization-entropy is conceptually more involved since entropy is inherently nonlocal in the sense that it cannot be obtained by a integrating a pointlike conserved current or any other operator but rather encodes a holistic aspect of an entire algebra. Nevertheless there is an algebraic analog of the above test function smearing: the splitting property [3].

Entropy in QM is an information theoretical concept which measures the degree of entanglement. The standard situation is bipartite spatial subdivision of a global system so that global pure states decompose into tensor product states and superpositions of product states called entangled. The entropy is than a number computed according in the well-known manner with the von Neumann prescription from the reduced impure state which results in the standard way from averaging over the opposite component.

The traditional quantum mechanical way to compute entanglement entropy was applied to QFT of a halfspace (a Rindler wedge in spacetime) for a system of free fields in a influential 1984 paper [61]. The starting point was the assumption that the total Hilbert space factorizes in that belonging to the halfspace QFT and its opposite. The calculation is ultraviolet divergent and after introducing a momentum space cutoff κ the authors showed that the cutoff dependence is consistent with an area behavior.

$$S/A = C\kappa^2 \tag{39}$$

where in the conformal case C is a constant and κ is a momentum space cutoff and S/A denotes the surface density of entropy. The method of computation is again the integration over the degrees of freedom of the complement region and the extraction of the entropy from the resulting reduced density matrix state whose degree of impurity encodes the measure of the inside/outside entanglement.

[41] Although the norm diverges, the inner product of $Q_R\left|0\right\rangle$ with localized states converges to zero in compliance with the zero charge of the vacuum.

It is easy to see this calculation in analogy to Heisenberg's calculation of charge polarization (37). In both cases the starting formula is morally correct but factually wrong. Neither is the partial charge inside a region defined by a volume integral nor do, as we know from previous sections, global states in QFT permit an inside/outside factorization. These incorrect assumptions create the divergencies which are then kept under the lid by QFT emergency aid: momentum space cutoff. In both cases dimensional arguments lead to an area proportionality. But the area appears only as a dimension-saving factor, there is no direct information that in both cases this behavior comes from vacuum polarization in a shell near the boundary and there is also no hint as to what is the correct formulation of the starting assumption. Whereas in the Heisenberg case the correct definition of the partial charge requires the test function formalism of pointlike currents, the algebraic counterpart in the case of entropy is the split property. With our preparation of this important concept in previous sections we now can plunge into medias res.

3.2 A modern point of view of localization entropy

Let us first apply the split idea to a two-dimensional conformal QFT in which case the double cone is a two-dimensional spacetime region consisting of the forward and backward causal shadow of a line of length L at $t=0$ sitting inside region obtained by augmenting the baseline on both sides by ΔL. As a result of the assumed conformal invariance of the theory, the canonical split algebra inherits the covariances and hence the entropy of the canonical split algebra can only be a function of the cross ratio of the 4 points characterizing the split inclusion

$$S = -tr\rho ln\rho = f(\frac{(d-a)\,(c-b)}{(b-a)\,(d-c)}) \tag{40}$$
$$with\ a < b < c < d = -L - \Delta L < -L < L < L + \Delta L$$

where for conceptual clarity we wrote the formula for generic position of 4 points. Our main interest is to determine the leading behavior of f in the limit $\Delta L \to 0$ which is the analog of the thermodynamic limit $V \to \infty$ for heat bath thermal systems.

The asymptotic estimate for $\Delta L \to 0$ can be carried out with an algebraic version of the *replica trick* which uses the cyclic orbifold construction in [62]. First we write the entropy in the form

$$S = -\frac{d}{dn} tr\rho^n|_{n=1},\ \rho \in M_{can} \subset \mathcal{A}(L+\Delta L) \tag{41}$$

Then one uses again the split property, this time to map the n-fold tensor product of $\mathcal{A}(L+\Delta L)$ into the algebra of the line (conveniently done in the compact S^1) with the help of the n^{th} root function $\sqrt[n]{z}$. The part which is invariant under the cyclic permutation of the n tensor factors defines the algebraic version [62] of the replica trick. The transformation properties under Moebius group are

now given in terms of the following subgroup of DiffS^1 written formally as

$$\sqrt[n]{\frac{\alpha z^n+\beta}{\bar{\beta}z^n+\bar{\alpha}}},\ L'_{\pm n}=\frac{1}{n}L_{\pm n},\ L'_0=L_0+\frac{n^2-1}{24n}c \tag{42}$$
$$\dim_{\min}=\frac{n^2-1}{24n}c$$

where the first line is the natural embedding of the n-fold covering of Moeb in diffS^{1}and the corresponding formula for the generators in terms of the Virasoro generators. As a consequence the minimal L_0' value (spin, anomalous dimension) is the one in the second line. With this additional information coming from representation theory we are able to determine at least the singular behavior of f for coalescing points $b\rightarrow a$, $d\rightarrow c$

$$S_{sing}=-lim_{n\rightarrow 1}\frac{d}{dn}\left[\frac{(d-a)(c-b)}{(b-a)(d-c)}\right]^{\frac{n^2-1}{24n}}=\frac{c}{12}ln\frac{(d-a)(c-b)}{(b-a)(d-c)} \tag{43}$$

Since the function is only defined at integer n, one needs to invoke Carlson's theorem.

The resulting entropy formula reads

$$S_{sing}=\frac{c}{12}\ln\frac{(d-a)(c-b)}{(b-a)(d-c)}=\frac{c}{12}ln\frac{L(L+\Delta L)}{(\Lambda L)^2} \tag{44}$$

where c in typical cases is the Virasoro constant (which appears also in the chiral holographic lightray projection).

This result was previously [63] obtained by the "inverse Unruh effect" for chiral theories which is a theorem stating that for a conformal QFT on a line the KMS state obtained by restricting the vacuum to the algebra of an interval is unitarily equivalent to a global heat bath temperature state at a certain geometry-dependent value of the temperature. The chiral inverse Unruh effect involves a change of length parametrization; the length proportionality of the heat bath entropy (the well known volume factor) is transformed into a logarithmic length measure.

Although the inverse Unruh effect is restricted to chiral theories, the analogy of the heat bath entropy with the localization entropy continues to exert itself. Below it will be shown that the localization entropy in the n-dimensional case diverges for $\Delta R\rightarrow 0$, with ΔR the splitting distance, as

$$E\overset{\Delta R\rightarrow 0}{\simeq}\frac{R^{n-2}}{(\Delta R)^{n-2}}ln\frac{R^2}{(\Delta R)^2} \tag{45}$$
$$V\simeq(\Delta R)^{n-2}\,ln\,(\Delta R)^{-2} \tag{46}$$

The reader will notice the close analogy to the heat bath entropy: the logarithm corresponds to the lightlike length factor of a lightlike slice of thickness ΔR and the inverse power is the analog of a transverse volume factor in the transformation from the thermodynamic limit to the funnel limit $\Delta R\rightarrow 0$ where the

second line expresses the correspondence between the heat bath volume factor and the divergence factors of the funnel limit of localization entropy.

Compared with the chiral models which can be controlled quite elegantly with the replica method, the question of higher dimensional localization entropy looks more involved. A closer look shows that the problem is not to identify the relevant density matrix leading to the localization entropy, but rather to explicitely compute its entropy and come up with a formula which replaces (39). The localization entropy associated with the double cone geometry may serve as the most typical illustration. To obtain a finite entropy one needs a sheet of finite thickness as a vault for the vacuum polarization this time in an algebraic form rather than test function smearing. For this purpose one uses the previously presented split property of two monads namely a smaller double cone algebra of size R inside a bigger (say symmetric around the origin)

$$\begin{aligned} &\mathcal{A}(\mathcal{D}(R)) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{D}(R+\Delta R)) \\ &\mathcal{A}(ring) \equiv \mathcal{A}(\mathcal{D}(R))' \cap \mathcal{A}(\mathcal{D}(R+\Delta R)), \\ &\mathcal{N} = \mathcal{A}(\mathcal{D}(R)) \vee J_{ring}\mathcal{A}(\mathcal{D}(R))J_{ring} \end{aligned} \tag{47}$$

where $\mathcal{N}$ is the canonically associated type I algebra in terms of which there is tensor factorization as in (35). The relative commutant in the second line is of special interest since geometrically it describes the finite shell region (or rather its causal completion) in which we expect the vacuum polarization to be localized in that ring. The restriction of the vacuum to $\mathcal{N}$ is a density matrix state ρ_{split} and the split entropy is the von Neumann entropy of this mixed state (there is a corresponding density matrix on $\mathcal{N}'$ which leads to the same entropy).

The only place where the split vacuum deviates significantly from the original vacuum is on observables in the ring region. This is the origin of the area proportionality (apart from a logarithmic correction)

The resulting formula is most clear in the conformal case because besides the length R which determines the hyperface "area" R^{n-2} the only other dimension carrying parameter is ΔR so that the entropy is

$$E = C(n)\frac{R^{n-2}}{(\Delta R)^{n-2}}\frac{c}{12}ln\frac{R(R+\Delta R)}{(\Delta R)^2},\ C(0)=1 \tag{48}$$

The physics of the case of the higher dimensional double cone entropy is similar since the leading contributions is given by the conformal limit. Note that the modular temperature is always fixed and generally different from the physical temperature. For the Unruh effect associated with the boost of a wedge region W the acceleration of the observer (which belongs to a whole family of observers) enters $T = 2\pi\frac{1}{a}$. In general the physical temperature differs from the modular by the "surface gravity".

The reason why we have preferred the double cone instead of the wedge region whose (modular group is geometric even in the massive case) which is in many aspects simpler is that the inclusion of two wedge algebras is not split.

The explanation is however very simple, the horizon is not finite since the transverse area is infinite and hence the would be density matrix resulting from the inclusion diverges.

3.3 Remarks on holography

The special role of null-surfaces as causal boundaries, which define places around which vacuum polarization clouds form, suggests that there may be more to expect if one only could make QFT on light-front a conceptually and mathematically valid concept. That this can be indeed achieved is the result of holography. Holography clarifies most of the problem which were raised by its predecessor, the "lightcone quantization" and explains why this method failed. One of the reasons has to do with short distance behavior since the naive restriction of fields to space- or light-like submanifolds require the validity of the canonical quantization formalism i.e. a short distance dimension not worse than sdd=1.

However the causal localization principle at least in its algebraic formulation permits to attach to each region the algebra of its causal shadow. For null-surfaces the situation is better. In that case the observable algebras indexed by regions on the lightfront are really field-generated and the field generators are transversely extended chiral fields $C(x, \mathbf{x})$ where x denotes the lightlike coordinate on the lightfront and $\mathbf{x}$ parametrizes the n-2 dimensional transverse submanifold. Their commutation relations are of the form

$$[C_i(x_1, \mathbf{x}_1), C_j(x_2, \mathbf{x}_2)] = \delta(\mathbf{x}_1 - \mathbf{x}_2) \sum_{k=0}^{m} \delta^{(k)}(x_1 - x_2) C_k(x_1, \mathbf{x}_1) \tag{49}$$

where the number m of operator contributions on the right depends on the scale dimensions of the two operators on the left hand side. As for standard chiral fields the scale dimensions are unlimited (no restriction to canonicity as for equal time commutations)[42]. The most useful and characteristic property of lightfront QFT is the total absence of transverse vacuum polarization, this is how the area behavior manifests itself in the lightfront generating fields.

The modular localization theory plays a crucial role in the construction of a local net on the lightfront and its generating fields and for this reason one must start with operator algebras which is in a standard position with respect to the vacuum. Since the full lightfront algebra is identical to the global algebra on Minkowski spacetime one must start with a subregion on the lightfront and the largest such region is half the lightfront whose causal completion is the wedge so that it can be seen as the wedge´s causal (upper) horizon $H(W)$[43]

$$\mathcal{A}(W) = \mathcal{A}(H(W)) \tag{50}$$

In the spirit explained in previous sections one the constructs the local structure of $\mathcal{A}(H(W))$ one intersects and recombines the W algebras which have their

[42] There can be higher derivatives in the transverse direction but they are always even whereas the light-like delta functions are odd.

[43] This is the quantum version of causal propagation with characteristic data on $H(W)$. A smaller region on LF does not cast a causal shadow.

horizons on the same lightfront. In 4-dimensional Minkowski spacetime they are connected by a 7-parametric subgroup of the 10-parametric Poincaré group containing: 5 transformations which leave W invariant (the boost, 1 lightlike translation, 2 transverse translations, 1 transverse rotation) and 2 which change W (the two "translations" in Wigner's Little Group). This is precisely the invariance group of the lightfront. It is not difficult to see that this net of observable algebras on the lightfront factorizes in the transverse direction. If this net has pointlike generators they are necessarily of the kind of transverse extended chiral fields (49).

For free fields the construction can be done explicitly. Since it is quite interesting and sheds some light on why the holography works whereas the lightcone quantization did not succeed the remainder of this section will present the free field holography.

The crucial property which permits a direct holographic projection is the mass shell representation of a free scalar field

$$A(x)=\frac{1}{(2\pi)^{\frac{3}{2}}}\int(e^{ipx}a^{*}(p)\frac{d^{3}p}{2p_{0}}+h.c.) \tag{51}$$

Using this representation one can directly pass to the lightfront by using lightfront adapted coordinates $x_{\pm}=x^{0}\pm x^{3},\ \mathbf{x}$, in which the lightfront limit $x_{-}=0$ can be taken without causing a divergence in the p-integration. Using a p-parametrization in terms of the wedge-related hyperbolic angle $\theta: p_{\pm}=p^{0}+p^{3}\simeq e^{\mp\theta},\ \mathbf{p}$ the $x_{-}=0$ restriction of $A(x)$

$$\begin{aligned}
&A_{LF}(x_{+},\mathbf{x})\simeq\int\left(e^{i(p_{-}(\theta)x_{+}+i\mathbf{px}}a^{*}(\theta,\mathbf{p})d\theta d\mathbf{p}+h.c.\right) \qquad (52)\\
&\left\langle\partial_{x_{+}}A_{LF}(x_{+},\mathbf{x})\partial_{x\prime_{+}}A_{LF}(x_{+}^{\prime},\mathbf{x}^{\prime})\right\rangle\simeq\frac{1}{\left(x_{+}-x_{+}^{\prime}+i\varepsilon\right)^{2}}\cdot\delta(\mathbf{x}-\mathbf{x}^{\prime})\\
&\left[\partial_{x_{+}}A_{LF}(x_{+},\mathbf{x}),\partial_{x\prime_{+}}A_{LF}(x_{+}^{\prime},\mathbf{x}^{\prime})\right]\simeq\delta^{\prime}(x_{+}-x_{+}^{\prime})\delta(\mathbf{x}-\mathbf{x}^{\prime})
\end{aligned}$$

The justification for this formal manipulation consists in using the fact that the equivalence class of test function which have the same restriction $\tilde{f}|_{H_{m}}$ to the mass hyperboloid of mass m is mapped to a unique test function f_{LF} on the lightfront [64][65]. It only takes the margin of a newspaper to verify the identity $A(f)=A(\{f\})=A_{LF}(f_{LF})$. But note also that this identity does not mean that the A_{LF} generator can be used in the bulk since the inversion involves an equivalence class and does not distinguish an individual test function in the bulk; in fact a localized test function $f(x_{+},\mathbf{x})$ is spread out in the bulk.

This corresponds to the classical causal shadow behavior of characteristic data on the light front: the causal shadow cast from half the lightfront is the associated wedge but the restriction to transverse or lightlike compact data does not improve the bulk localization i.e. the sub $H(W)$ localization does not improve the bulk localization, it only causes fuzziness. So algebraic holography from a wedge in the bulk is not invertible. the local substructure of a wedge

algebra $\mathcal{A}(W)$ cannot be fully encoded into $\mathcal{A}(H(W))$, although the two global algebras are identical. This also applies to event horizons in curved spacetime and is incompatible with the idea that the information contained in the local bulk substructure of a region can be encoded into its horizon (for more remarks see the conclusions).

For the case at hand namely the bulk- and lightfront- generators this projective nature of holography asserts itself in the fact one cannot reconstruct from the space of $H(W)$ localized smearing functions the local substructure of the space of W-bulk localized test functions. The projection can be upgraded to an isomorphism by injecting additional knowledge e.g. knowledge about how the Poincaré transformation which are not part of the 7-parametric group act on the lightfront generators. This is a trivial step if the generators of the holographic projection in case their Poincaré covariance is known as in the above case of the $a(p), a^*(p)$ annihilation/creation operators; one only has to apply the x_- translation in order to reconstitute the original bulk generators. It can be shown that under certain reasonable assumptions of a rather general nature the full bulk structure can be recovered from knowing the local net of holographic lightfront projections in different lightfront positions related by Poincaré transformations.

Historically the "lightcone quantization" which preceded lightfront holography shares with the latter part of the motivation namely the idea that by using lightlike directions one can simplify certain aspects of an interacting QFT. But as the terminology "quantization" reveals this was unfortunately mixed up with the erroneous idea that in order to achieve this one needs a new quantization instead of a radical spacetime reordering of a given abstract algebraic operator substrate whose Hilbert space is always maintained. As often such views about QFT results from an insufficient appreciation of the autonomy of the causal locality principle by not separating it sufficiently from the contingency of pointlike fields.

Formally mass shell representations also exist for interacting fields. In fact they appeared shortly after the formulation of LSZ scattering theory and they were introduced in a paper by Glaser, Lehmann and Zimmermann and became known under their short name of "GLZ representations". They express the interacting Heisenberg field as a power series in incoming (outgoing) free fields. In case there is only one type of particles one has:

$$A(x) = \sum \frac{1}{n!} \int_{V_m} \cdots \int a(p_1, ...p_n) e^{i \sum p_k x} : A_{in}(p_1)...A_{in}(p_n) : \frac{d^3 p_1}{2p_{10}} ... \frac{d^3 p_1}{2p_{10}} \quad (53)$$

$$A_{in}(p) = a_{in}^*(p) \text{ on } V_m^+ \text{ and } a_{in}(p) \text{ on } V_m^-$$

$$a(p_1, ...p_n)_{p_i \in V_m^+} = \langle \Omega | A(0) | p_1, ...p_n \rangle \quad (54)$$

where the integration extends over the forward and backward mass shell $V_m^{\pm} \subset V_m$ and the product is Wick ordered. The coefficient functions for all momenta on the forward mass shell V_m^+ are the vacuum polarization components of A and the various formfactors (matrix elements between in ket and out bra states) of are believed (the crossing property) to be mass shell boundary values of Fourier-

transformed retarded functions.

The convergence status of these series is unknown[44], but it is evident that the formal lightfront restriction for each term in (53) does not cause any short distance divergence. It is also clear that it is not possible to define a lightfront restriction on vacuum expectations (Wightman functions), one really needs to reconstruct the operators and verify the prerequisites for a mass shell representations as (53). In contrast to the algebraic setting the holography based on the GLZ formula is inherently nonlocal since it requires the full insight into the nonlocal relation between interacting and incoming fields. There is however no restriction on the short-distance dimensions of the fields as there was in the old "lightcone quantization.

The holography of individual fields in the mass shell representation highlights some interesting problems which are important for autonomous nonperturbative constructions of models in QFT of the kind i.e. constructions which do not depend on Lagrangian quantization as those presented after (26) . The more rigorous algebraic method by its very nature (using relative commutants) only leads to bosonic holographic projections. This means that the extended chiral structure on the lightfront only contains integral values in its short distance spectrum; i.e. the generating fields are of the kind of the chiral components of two-dimensional conserved currents and energy-momentum tensors. Hence only a small subalgebra of the bulk algebra[45] associated with transverse extended currents, energy momentum tensor etc. will appear; there would be no anomalous dimension field in the algebraic holographic projection.

The obvious conjecture is that the objects belonging to the anomalous dimensional spectrum which could not pass through the "algebraic holographic projection filter" can be reconstructed via representation theory of (extended) chiral observable algebras, a version of the DHR superselection theory which is particularly well developed in chiral models. The pointlike field holography based on the mass shell representation supports this idea that the anomalous dimensions of bosonic bulk fields become holographically encoded into the spin-statistics and scale dimensions of plektonic (anyonic in the abelian case) chiral fields. Again the projection carries a lot of information about the bulk but holographic data on one horizon alone do not allow a unique inversion i.e. holography on null-surfaces does not lead to an isomorphism. If on the other hand one would know a GLZ-like representations of the generating lightfront fields one can obtain the GLZ representations of the bulk fields simply by a x_- translation.

Clearly many of these ideas, as important for the future development of QFT as they may appear, are not yet mature in the sense of mathematical physics. Therefore it is good to know that there exists an excellent theoretical laboratory to test these ideas in a better controlled mathematical setting:: two-dimensional factorizing models and their this time bona fide (no transverse extension) chiral holographic projection. From a previous section on modular theory we know

[44]In contrast to the perturbative expansion which is known to diverge even in the Borel sense, the convergence status of GLZ had not been settled.

[45]Apart from conserved currents whose charges must be dimensionless, fields are not protected against carrying non-integer short distance scale dimensions.

that these models have rather simple on-shell wedge generators $Z(x)$ which still maintain a lot of similarity with free fields. In that case Zamolodchikov proposed a consistency argument which led to interesting constructive conjectures about relations between factorizing models and their critical universality classes represented in form of their conformal short distance limits.

Conceptual-wise the critical conformal limit is very different from its holographic projection, the former is a different theory whose Hilbert space has to be reconstructed from the massless correlation function whereas the latter is just a reprocessing of spacetime ordering of the original quantum substrate in the original Hilbert space. Assuming that one knows the chiral fields on the lightray as a power series in term of the Z-operators[46] one has a unique inversion, i.e. the holographic projection becomes an isomorphism.

Calculations on two models [46], the Ising field and the Sinh-Gordon field, have shown that the universality class method and the holographic projection lead to identical results[47]. Whereas the anomalous dimension of the sinh-Gordon field can not be computed approximately in terms of doing the integrals in the lowest terms in the mass shell contributions, the series for the Ising order field can be summed exactly and yields the expected number $1/16$. This is highly suggestive for reinterpreting the Zamolodchikov way of relating factorizing models with chiral models as part of holographic projection which may be used as a rigorous relation between quantum matter in the bulk and its spacetime reordered presentation on the lightray horizon.

The gain in modular symmetry is perhaps the most intriguing aspect of holography. In general the modular theory for subwedge localization of bulk lead to algebraic modular groups which cannot be encoded into diffeomeophisms of the underlying spacetime manifold; the generators of these groups are at best pseudo-differential operators. However there are strong indications that their restriction to the horizon are geometric. This situation is particularly interesting in generic spacetime manifolds which have no bulk symmetry.

This reinterpretation emphasizes the role which holography is expected to play in the future development of QFT: introduce a different viewpoint about QFT which permits to partition the difficult task to construct interacting models into many less difficult tasks. Certainly chiral models are simpler than any other model, in fact the classification in terms of families and their explicit construction has already progressed [79].

Since our presentations of localization entropy and lightfront holography was in the setting of Minkowski spacetime where there are only causal horizons but no event horizons the question arises whether there is any reason to expect any change on black hole event horizons. In view of speculations about black hole physics in the literature a more specific question would be is it conceivable that the holography onto the black hole event horizon becomes an isomorphism instead of a projection so that the whole world above the horizon becomes

[46]From the point of view of chiral models such a representation is of course somewhat unusual.

[47]The consistency of the holographic lightray projection with the critical limit for factorizing models was checked in an oral discussion with Michael Karowski..

imaged onto the horizon? This sounds a bit like science fiction, after all the Kruskal extension of the Schwarzschild black hole is of the same bifurcated kind as the wedge situation. There is really no support for such an idea from QFT on event horizons or from speculations about QG unless one books the lack of knowledge as an asset for a speculative idea.

The impossibility to store all information in the bulk into a horizon can already be seen from what is known about the classical characteristic value problem i.e. the Cauchy problem on null-surfaces as the lightfront. Whereas from the local data on the lightfront it is possible to reconstruct the data on certain semiinfinite regions in the bulk as lightlike slabs with either infinite extension in lightlike- or spacelike transverse direction, it is not possible to do this for compactly extended bulk data. This has its precise analog in the quantum case where the local substructure on the lightfront can only retrieve the operator algebras indexed by the mentioned semiinfinite regions. In order to recover the full net of spacetime indexed subalgebras one either needs to know the action of those Poincaré symmetries beyond the 7-parametric symmetry subgroup of the lightfront or (in case of CST without symmetries) the data on more than only one null-surface. An holographic isomorphism in which the lower dimensional manifold has to carry the burden of more than the cardinality of degrees of freedom as it would be natural for that lower spacetime dimension only happens in case of the AdS-CFT correspondence [36].

3.4 Vacuum fluctuations and the cosmological constant problem

If there is any calculation which holds the record for predicting a quantity which comes out way off the astrophysically observed mark, namely by at least 40 orders of magnitudes, it is the estimate for the cosmological constant based on a quantum mechanical argument of filling particle levels above the vacuum in a similar spirit as occupying levels up to the Fermi surface for obtaining the ground state for many body systems at finite density and zero temperature[48]. As pointed out by Hollands and Wald [18] such global occupation arguments for computing a local density contradict the holistic aspect of global reference states in a theory which fulfills the global covariance principle. the vacuum So the estimate which led to this a gigantic mismatch between quantum mechanics of free relativistic particle and the astrophysical reality has no credibility. A "cosmological constant problem" in the sense of mismatch between particle theory and cosmological observation does not exist and arguments which have been designed to find a way out of this problem as e.g. the invocation of an *anthropic principle* share this lack of credibility..

It is instructive to look first at the problem of the cosmological energy density i.e. the expectation value of the zero-zero component of the energy-stress tensor

[48]The estimate consists in in filling free energy levels above the free vacuum state up to a certain cutoff mass κ which should be larger than all the physical masses and smaller than the mass corresponding to the Planck length. The result of such a calculation leads to an energy density $\rho_E \sim \kappa^4$.

in a state φ in a QFT in CST. The standard argument by which one defines the stress-energy tensor as a composite of a field is well known for free fields, one starts from the bilocal split-point expression and takes the coalescing point limit after subtracting the vacuum expectation value so that the result agrees with the Wick-ordered product. The resulting stress-energy tensor has all the required properties. Its expectation value is well-defined on a dense set of states which includes the finite energy states. But contrary to its classical counterpart, there is a (unexpected at the time of its discovery [75]) problem with its boundedness from below since one can find state vectors on which the energy density $T_{00}(x)$ takes on arbitrarily large negative values.

This had of course led to worries since classical the positivity inequalities were known to be crucial for questions of stability. It started a flurry of investigations [77] which led to state-independent lower bounds for fixed test functions $T_{00}(f)$ as well as inequalities on subspaces of test functions. These inequalities which involve the free stress-energy tensor were then generalized to curved space time[49]. In the presence of curvature the main problem is that the definition of $T_{\mu\nu}(x)$ is not obvious since in a generic spacetime there is no vacuum like state which is distinguished by its high symmetry; and to play that split point game with an arbitrarily chosen state will not produce a locally covariant energy stress tensor.

A strategy to do this was given in 1994 by Wald [76] in the setting of free fields. His postulates gave rise to what is nowadays referred to as the *local covariance principle* which is a very nontrivial implementation of Einstein's classical covariance principle of GR to quantum matter in curved spacetime (after freeing the classical principle from its physically empty coordinate invariance interpretation). determines the correct energy-momentum tensor up to local curvature terms (whose degree depends on the spin of the free fields). To formulate it one needs to consider all Lorentz mannifolds with a certain causality structure simultaneously [35].

In fact one can construct a basis of composite fields so that every member is a locally covariant composite of the free field such that for the Minkowski spacetime we re-obtain the simpler Wick basis. The formulation of the local covariance principle uses local isometric diffeomorphisms of the kind which already appeared in Einstein's classical formulation and this requires to consider simultaneously all QFT which share the same quantum substrate but follow different spacetime ordering principles. In other words, even if one's interest is to study QFT in a particular spacetime (Robertson-Walker for the rest of this section), one is forced to look at all globally hyperbolic spacetimes in order to find the most restrictive condition imposed by the local covariance principle.

The result is somewhat surprising in that this principle cannot be implemented by taking the coincidence limit after subtracting the expectation in one of the states of the theory. Rather one needs to subtract a "Hadamard parametrix" [34] i.e. a function which depends on a pair of coordinates and is defined in geometric terms; in the limit of coalescence it depends only on

[49] For recent publication with many references see [78].

the metric in a neighborhood of the point of coalescence. Only then the global dependence on the metric carried by states can be eliminated in favor of a local covariant dependence on $g_{\mu\nu}(x)$ and its derivatives. As a result the so-constructed stress-energy tensor at the point x depends only on the metric in an infinitesimal neighborhood of x.

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Recently these renormalization ideas were applied to computations of back-reactions of a scalar massive free quantum field in a spatially flat Robertson-Walker model. As a substitute for a vacuum state one uses a state of the Hadamard form since these states fulfill a the so-called microlocal spectrum condition which emulates the spectrum condition in Minkowski spacetime. The singular part of a Hadamard state is determined by the geometry of spacetime. The renormalization requirements of Wald lead to a an energy momentum tensor with 2 free parameters which can be conveniently represented as functional derivatives with respect to the metric of the two quadratic invariants which one can form from the Ricci tensor and its trace. In [19] the resulting background equations were analyzed in the simpler conformal limit and it was found that the quantum backreaction stabilizes solutions i.e. accomplishes a task which usually is ascribed to the phenomenological cosmological constant. Without the simplifying assumption the linear dependence on a free renormalization parameter guaranties that any measured value can be fitted to this backreaction computation. The principles of QFT cannot determine renormalization parameters.

Hence from a QFT point of view there is no cosmological problem which places QFT in contradiction with astrophysical observations. A consistency check would only be possible if there are other measurable astrophysical quantities which fall into the setting of quantum backreaction on spatially flat RW cosmologies.

4 Resumé, additional comments and outlook

The backbone of this essay has been the quantum counterpart of theories with a maximal velocity[50] as compared to those without. This leads to very different QTs coming with a radically different localization concept: B-N-W localization related to particles and modular (causal) localization implemented by local observables. The dividing line, as the existence of the macrocausal DPI setting shows, is not special relativity per se but more specifically the (micro)causal nature of the interaction following from the existence of a maximal velocity.

Both DPI and QFT can be formulated with a joint starting point namely the one-particle representation spaces as classified by Wigner and their multiparticle tensor products. Whereas DPI introduces interactions by modifying the noninteracting n-particle representation "by hand" in such a way that the

[50]Note that in theories with unbounded velocities, special situations may lead to effective maximal velocities (e.g. velocity of sound, relativistic particle propagation in DPI).

modifications of the Poincaré generators for different n are tied to each other by cluster factorization, the path from Wigner to QFT is only simple in the absence of interactions when there is a functorial relation between modular localized subspaces of the one-particle Wigner space and local operator subalgebras in a bosonic or fermionic Wigner Fock space. Free fields display themselves as pointlike coordinatizations of these algebras i.e. as singular (operator-valued distributions) generators of the net of spacetime indexed algebras. Whereas the localized one-particle states and the system of local algebras are unique, there is a countable infinite plurality of relative local pointlike field coordinatizations which can be divided into two groups, generators which are linear in Wigner particle creation and annihilation operators and composites thereof i.e. the Wick ordered monomials of the linear generators. One can also use stringlike generators (and in some cases there are no pointlike fields) of which there exist continuously many. The traditional way to introduce interactions in accord with the locality principle is by coupling these generating fields "by hand" which is supported by the non-intrinsic Lagrangian quantization formalism. A truely intrinsic nonperturbative approach based on the classification of generators for wedge algebras exists presently only for a subfamily of two-dimensional factorizing models.

Whereas the only localization in the DPI setting is that of B-N-W localized wave functions in terms of a position operator and its spectral projectors, the modular localization in LQP leads to dense subspaces and causally complete subalgebras. Dense subspaces which change with the spacetime region do not fit into the standard setting of QT in which single operators and projectors on subspaces play the prominent role. Hence it does not come as a complete surprise that the literature on this particle-field issue is unfortunately also somewhat confusing. For many decades QFT was viewed as being part of the same conceptual setting as QM and only more recently a perception of their substancial differences developed.

Often deep antagonisms were construed which are really not there. Particles and fields are in a very precise way asymptotically connected and the B-N-W localization leads to covariant scattering probabilities for particles precisely where it is needed, namely for the asymptotic relation[51] and whether the coincidence and anticoincidence counters measure asymptotic B-N-W localization or modular localization is somewhat academic. Instead of dwelling on the lack of covariance of the B-N-W particle localization it is more realistic to take the proverbial point of view of the half full glass and emphasize its *asymptotic* covariance. Without the asymptotic particle concept there would be no *stability* and *objectivity* since fields are, like coordinates in geometry, and hence one would not know for sure which one is being measured. Unlike in classical field theory, where fields have "individuality" (e.g. electromagnetism), the main role of quantum fields is to "interpolate" particles and at the some time to implement principles which cannot be formulated in terms of particles and their

[51]Whereas the relation between B-N-W localized events are only *effectively* covariant at distances larger than a Compton wave length of the to be localized particles, events seperated by an infinite timelike distance lead to invariant transition probabilities.

S-matrix. Although there exists an infinite equivalence class of interpolating fields, they interpolate the same situation; if one wants to avoid the plurality of fields altogether one can also interpolate the unique system of particles directly with the unique system of local algebras [73].

Part of the particle-field muddle comes from placing Lagrangian quantization into the center of QFT and extracting conceptual messages from the proximity of the Lagrangian quantization formalisms of QM and QFT. This puts a strain on separating genuinely intrinsic physical properties from those which are contingent on a particular computational scheme; and it often takes a lot of thinking to arrive at the conclusion, that besides spacetime-indexed local algebras on the local level and asymptotic particle states and their scattering probabilities, there are no intrinsic concepts in QFT; at the end of the day the classification and construction of models of QFT have to be understood in terms of these principles and the Lagrangian formalism is only a temporary crutch.

Without the asymptotic probabilities which only enter through B-N-W localization, particle physics would not be what it is. QFT in CST in generic spacetimes (without timelike Killing vectors) lacks these particle concept; as a consequence of absence of the necessary spacetime symmetries there is no distinguished vacuum reference state. The question of what remains of particle physics if a model of QFT admits no standard particle aspects is a very serious one even in the context of Minkowski space QFT [29].

In generic curved spacetime the prerequisite of Poincaré group representation theory is absent so that in additions to particles even the vacuum state has disappeared. This raises the question of what, after measurements of particles and their scattering cross sections have disappeared, what remains to be measured at all, is it fields? Certainly there are still some radiation densities which can be measured in form of the Hawking/Unruh radiation or cosmic background radiation, but the rich scattering theory, i.e. particle physics as we have known it for almost 8 decades, does not seem to have a CST counterpart. Wald [74] has argued that one should think in terms of measuring fields, but it is not clear what this means since fields are of a fleeting nature and their stable non-fleeting aspect consists precisely in their particle/infraparticle content. In any case the question of what becomes of particles in QFT in CST is an important open problem.

A large part of this essay was used to expose the existence of two different kind of *entanglements* which results from a spatial bipartite division of the global algebra. The standard entanglement picture of quantum information theory applies only to a quantum mechanical B-N-W localized subalgebra and its commutant which is B-N-W localized in the complement region; the result is a tensor factorization and the ensuing notion of entanglement is that one studied within (quantum) information theory.

In causal QFT such a bipartite division automatically involves the causal completion of the spatial compact region and its causal disjoint which is the causal completion of the spatial complement. In this case there is no tensor factorization and any attempt to go ahead as if it existed leads to divergent expression; the integrals can be made finite by the use of the practitioner's

sledge hammer: a momentum space cutoff[52]; but as usual this does not provide any insight about what is really going on. With some hindsight one may conclude from such carefully executed cutoff calculations [61] the leading term of an vacuum polarization-caused area law, but the full insight only results from answering the question why the tensor factorization fails in the first place.

As well-known the restriction of globally pure state (vacuum, particle states) to causally localized subalgebras $\mathcal{A}(\mathcal{O})$ leads to thermal KMS states associated with the modular Hamiltonian associated to $(\mathcal{A}(\mathcal{O}), \Omega)$. Modular Hamiltonians give rarely rise to geometric movements (diffeomorphisms). Although in Minkowski spacetime there is no compact localization region which leads to a *geometric* modular theory, such situations do occur in connection with appropriate Killing symmetries in CST if one restricts suitable global states to a black hole region[53].

Although the terminology "entanglement" strictly speaking does not apply to a bipartite separation with sharp causal boundaries, the literature on entanglement unfortunately does not differentiate between the QM and the QFT case (mostly unknowingly, but sometimes also knowingly [16]).

We have seen that the split property permits a tensor factorization in which one tensor factor contains the causally localized algebra, and the second tensor factor contains its causal disjoint after splitting it away from spatially touching the original algebra. There is an external parameter entering the factorization based on the splitting method, namely the splitting distance ΔR. The restriction of the global vacuum to one of the tensor factors is a Gibbs state at a temperature which depends on the normalization of the modular Hamiltonian which is uniquely associated with this situation. The entropy of this Gibbs state diverges with decreasing split $\Delta R \to 0$ and this explains the divergence of the momentum space cutoff and shows that, different from other divergences in QFT, the localization entropy in the limit of sharp localization is not a result of a pathology of the theory or in its formulation but rather the hallmark of a causal QFT with or without CST. Note that although the split property paves the way for a return of the entanglement concepts, the homecoming is not complete since the localization-entanglement is thermal and not information theoretical; there is simply no way in which a QM bipartite situation can have a thermal entanglement without having been globally thermal from the start.

The area law and the divergence in the zero split limit hold for localization with causal horizons as well as for localization behind curved spacetime-related event-horizons as they occur in black hole physics. This requires a revision of what is thought to be the interface between QFT in CST and QG. In many articles the Bekenstein area law (after its quantum reinterpretation as a property

[52] Whereas the divergencies in perturbation theories are not intrinsic i.e. can be avoided by a more appropriate formulation, the divergencies due to vacuum polarization at the locus of causal/event horizons are genuine properties of causal QFT which cannot be "renormalized away".

[53] Example: the Hartle-Hawking state on the Kruskal extension restricted to the region outside a black hole.

of entropy) was hailed as being part of this interface since it was not recognized that the *area behavior is a totally generic behavior of local quantum physics*. To claim that if not the area law per se than at least Bekenstein's specific gravitational dependent value is a property of the still illusive QG has a certain plausibility although it would be the fist time in the history of QT that a classical constant does not require a quantum modification.

The Bekenstein thermodynamical interpretation of a certain quantity in the setting of classical gravity raises the question whether it is not possible to invert this connection i.e. to supplement the thermodynamical setting by reasonable assumptions of a general geometric nature so that the Einstein Hilbert equations are a consequence of the fundamental laws of thermodynamics. Modular theory already relates thermal behavior with localization, hence a relation of fundamental laws of thermodynamics with gravity is not as unexpected as it looks at first sight. The reader is referred to some very interesting observations by Jacobson [72].

Another property attributed to QG is that the event horizon stores a complete image of the bulk world, i.e. the holography is really an isomorphism. There are cases in QFT where holography onto a boundary becomes an isomorphism (viz. the AdS-CFT correspondence) but certainly not on horizons which are null-surfaces. In the latter case the degrees of freedom on the horizon are always of a lesser cardinality than those in the bulk and only by enlarging them by spacetime transformed degrees of freedom outside the null surface can one return to the bulk. The idea that of a holographic image of the world may in a future QG setting turning into an isomorphism enjoys some popularity does not sound very palatable, but it is difficult to criticize something for which no arguments are given.

As we have seen there is a sharp dichotomy between quantum mechanical *information theoretical entanglement* and the *thermal entanglement* resulting from modular localization. Hence it is unclear what the *black hole information loss* means in the setting of a thermal localization. In many of the articles the terminology QM instead of QFT is used, thus making it obvious that the authors do not appreciate the fundamental differences in the notion of entanglement between QM and QFT.

One of the great advances in reconciling QFT and general relativity is the discovery of the quantum counterpart of local covariance whose implementation requires to spacetime-organize an abstract algebraic substrate (e.g. CCR- CAR-algebra) simultaneously on all possible globally hyperbolic manifolds together[54] (including of course the Minkowski spacetime) so that the algebraic substrate on isometrically related manifolds is isomorphic. Since isomorphic situations cannot be distinguished by experiments within their localization region, the local covariance principle accomplishes the realization of an important aspect of *background independence*.

The partisans of quantum gravity think that such situations should not only be isometric but even identical. In the previous section we have seen that the

[54]Each causally complete submanifold is also an admissible manifold.

local covariance setting has led to the first calculations involving backreactions in cosmological situations. This makes it possible to address the problem of the cosmological terms as having its origin in "vacuum" energy where vacuum in this contest is the euphemistic name for an unknown cosmic reference state.

Perhaps the most profound difference between QM and QFT finds its expression in the encoding of a finite number of monads into a certain "modular" position within a joint Hilbert space. Concretely one thinks of a finite collection of wedge algebras in certain geometric positions which correspond to nontrivial modular inclusions and intersections. But the modular positioning is intrinsic and abstract and in particular does not directly refer to spacetime and its Poincaré invariance group. Rather the latter together with a spacetime-indexed local net of operator algebras is derived from a special kind of modular positioning. Even the unique nature of the operator algebras of being monads i.e. hyperfinite type III_1 algebras is a consequence since only field theoretic monads allow this positioning. As mentioned in the section on modular positioning there have been other ideas to highlight the relational nature of QT in particular Mermin's view of QM in terms of its correlations as expressed by his apodiction:

Correlations have physical reality, that what they correlate does not.

We may express the relational nature of LOP as resulting from modular positioning as:

Relative modular positions in Hilbert space have physical reality, the quantum matter they position does not.

The presentation of QFT in terms of positioning monads is very specific of LOP i.e. it has no analog in QM i.e. Mermin's view is not a special case of positioning in LQP.

Philosophically distinctive viewpoints are however not always the most appropriate ones for actual constructions. Indeed knowing the action of the Poincaré group on one monad (interpreted as a wedge algebra) instead of the modular positioning of several is the more practical starting point. The most efficient way to characterize a wedge subalgebra $\mathcal{A}(W) \subset B(H)$ is in terms of generators. As explained in the paper, in factorizing two-dimensional models simple generators are known, under suitable conditions they are Fourier transforms of Zamolodchikov-Faddeev creation/annihilation operators. In those cases the algebraic construction leads to nontrivial double cone algebras and finally to the first existence proof of models which have worse short distance behavior than that allowed by canonical commutation relation.

One would hope for more along these algebraic lines but in view of the fact that this is the first existence proof in the almost 80 years history of QFT it should be very encouraging and provide a strong motivation for continuing along these lines.

There have been similar proposals along modular lines, the most prominent one being the condition of geometric modular action (CGMA) [81] in which the modular conjugations rather than the modular groups play the important generating role. Although they have not been used for the constructions of models, they proved very useful in the clarification of structural properties notably the relation of spacetime symmetries with respect to inner unbroken or

spontaneously broken symmetries.

Being interested in the interface between QFT in CST and the still illusive QG it is natural to ask whether the characterization of QFT in terms of modular positioning of monads extends beyond Minkowski spacetime. For the simplest nontrivial kind of QFT, namely chiral theories on a circle, it is well-known that the Moebius symmetry follows from the modular positioning of two monads, but it does not lead to more general diffeomorphisms of which none leaves the Moebius-invariant vacuum fixed. The second message comes from the representation theory of the Virasoro algebra and states that there can be no vector at all which is left invariant under a higher diffeomorphism.

Consider for example a diffeomorphism with 4 equally distributed fixed points. Its geometric aspect leads one to expect a relation with the modular theory of a 2-interval. It turns out that the modular group of such algebras act in the expected way as the diffeomorphism with the 4 fix points, but it does do only on the two interval algebra and not on its complement where its action remains "fuzzy" i.e. not describable in terms of diffeomorphism. So the message is that if one admits appropriate vectors which change with the geometrical situation ("adjusted vacua") and studies the modular theory one can build up higher diffeomorphism on suitable multi-intervals. The fact that the diffeomorphism *coalesces with a modular group only on a multi-interval* is no hindrance.

In view of the nature of local covariance principle such "partial" local diffeomorphisms together with "partial vacua" seem to be a natural local generalization of the global vacuum and its associated global symmetries. Without pressing ahead with the modular positioning approach and reach its limits, one probably has no chance to get to the interface between QFT in CST and QG. Whenever one thought to have the first glimpse at QG, as in the example of the entropic area law or the principle of independence on the background, one found something in the already existing QFT in CST which put this view into question. In the case of entropy it was the general area proportionality, and in the case of background independence the isomorphism between causally closed parts of different worlds which are diffeomorphic as manifolds[55]. Thus whenever one deemed to finally have localized the interface between QFT in CST and QG it volatilized again.

On the other hand there is hardly any doubt that the QM-QFT interface had reached its conceptual final position. Apart from cosmetic changes one does not expect major conceptual relocation, even if there remains still a lot of refurbishing for the quantum measurement and philosophy of science communities.

After completing this essay I became aware of the existence of two papers by Steve Summers [82][83] where among other things different consequences of the split property concerning the localization of spacetime and inner symmetries are presented. Both papers are a rich source for additional references.

[55]It is not clear whether the stronger form of background independence, in which the isomorphism is replaced by an identity, can be achieved.

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